Homework 4

Math 302 (section 501), Fall 2016

This homework is due on Thursday, September 22.

0. *(This problem is not to be turned in.)*
   (a) Read Section 3.2.
   (b) (Practice Problems) Section 3.2 # 2, 3, 14, 25, 26, 30, 36, 45, 46
1. Read Timo De Wolff's *How to Present Homework Solutions*, available here: `http://www.math.tamu.edu/~dewolff/Spring16/How_to_do_a_Homework.pdf` What did you find interesting, surprising, or useful?
2. Show that the following statements hold (i.e., give witnesses and show the required inequality).
   (a) $f(x) = x^2 + 14x + 5$ is $O(x^2)$.
   (b) $f(x) = x^2 - 15$ is $\Omega(x)$.
   (c) $f(x) = 2x \cdot \log(x)$ is $O(x^2)$ (you may use that $\log(x) < x$ for every $x > 0$).
   (d) $f(x) = x^3 + 5x^2 - 5x$ is $\Theta(x^3)$.
3. Let $f_k(n) : \mathbb{Z}^+ \to \mathbb{Z}^+$ be the function given by $n \mapsto 1^k + 2^k + \cdots + n^k$ for some given positive integer $k > 1$. Show that $f_k(n)$ is $O(n^{k+1})$.
4. Let f and g be functions (from $\mathbb{R}$, $\mathbb{Q}$, or $\mathbb{Z}$ to $\mathbb{R}$, $\mathbb{Q}$, or $\mathbb{Z}$). Show: If $f(x)$ is $O(g(x))$, then $g(x)$ is $\Omega(f(x))$.
5. (Bonus problem – optional!) Let $g_r(n) : \mathbb{Z}^+ \to \mathbb{Z}^+$ be the function, which is for every positive integer $r > 1$ given by

$$\begin{array}{rcl} g_r(n) & = & 1 + 1^2 + \cdots + 1^r \\ & + & 2 + 2^2 + \cdots + 2^r \\ & \vdots & \vdots \\ & + & n + n^2 + \cdots + n^r. \end{array}$$

   Show that $g_r(n)$ is $O(n^{r+1})$.

   Hint: Let for every $k > 1$ $f_k(x)$ be defined as in Exercise 3. Express $g_r(x)$ as sum of $f_k(x)$ for suitable k.
6. Section 3.2. # 8, 17, 19, 37