Homework 12

Math 415 (section 502), Fall 2015

This homework is due on Thursday, November 19. You may cite results from class or previous homeworks/exams.

0. *(This problem is not to be turned in.)*
   (a) Read Section 19.
   (b) Section 18 #22, 48 (this *subring criterion* builds on the earlier *subgroup criterion*), 49
   (c) Section 19 #18, 24, 27
1. True/false (No proofs necessary for this problem.)
   (a) If R and R' are any two rings, then there exists a ring homomorphism $R \to R'$.
   (b) The rings $\mathbb{Z}$ and $\mathbb{R}$ are isomorphic (as rings).
   (c) $\mathbb{Z}_8^*$ (the group of units of $\mathbb{Z}_8$) is isomorphic to the Klein 4-group.
   (d) $\mathbb{Z}_{10}^*$ is isomorphic to the Klein 4-group.
   (e) $\mathbb{Z}_9^*$ is isomorphic to the Klein 4-group.
2. For each function below, determine whether it is a ring homomorphism. Give a proof.
   (a) $\phi : \mathbb{Z} \to \mathbb{Z}_{10}$ given by $n \mapsto 6n$
   (b) $\psi : \mathbb{Z} \to \mathbb{Z}_{10}$ given by $n \mapsto 5n$
   (c) $\gamma : \{\text{functions } \mathbb{R} \to \mathbb{R}\} \to \mathbb{R}$ given by $f \mapsto f(5) - f(6)$.
3. What are the units of $\mathbb{Z} \times \mathbb{Q} \times \mathbb{Z}_4$? What are the zero divisors? (No proof necessary.)
4. In $M_2(\mathbb{Z})$, is $\left(\begin{smallmatrix} 1 & 0 \\ -2 & 0 \end{smallmatrix}\right)$ a unit? Is $\left(\begin{smallmatrix} 1 & 1 \\ 0 & 2 \end{smallmatrix}\right)$? Is either a zero divisor? (Explain your answers.)
5. (a) Are $\mathbb{Z}$ and $2\mathbb{Z}$ isomorphic rings? Give a proof.
   (b) Are $2\mathbb{Z}$ and $5\mathbb{Z}$ isomorphic rings? Give a proof.
6. Read about the *characteristic* of a ring (pages 181–182).
   (a) Define the characteristic.
   (b) State an equivalent definition for rings with unity.
   (c) Determine the characteristics of each of the following rings: $\mathbb{Z} \times \mathbb{Q}$, $\mathbb{Z}_5[x] :=$ {polynomials with coefficients in $\mathbb{Z}_5$}, $2\mathbb{Z}$, and $M_n(\mathbb{Z})$. (No proof necessary, but show your work.)
7. Section 19 #17, 26, 29; Section 18 #50 (you may use the subring criterion from #48)