Homework 5

Math 415 (section 502), Fall 2015

This homework is due on Thursday, October 1. You may cite results from class or previous homework, as appropriate.

0. *(This problem is not to be turned in.)*
   (a) Read Sections 8–9.
   (b) Section 8 # 10, 19, 36
   (c) Give an example of a non-cyclic group for which all of its proper subgroups are cyclic.
   (d) Explain in your own words what a *finitely generated* group (or subgroup) is.
   (e) Is $\mathbb{R}$ a finitely generated group?
1. Prove that if a group G has finitely many subgroups, then G is a finite group.
2. (No proofs necessary for this problem, but show your work.)
   (a) Draw the Cayley digraph for $\mathbb{Z}_8$ that comes from the generating set $S = \{2, 5\}$.
   (b) Compute the order of the following permutation (which is written in 2-line notation):
   $$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix} \in S_5 .$$
   (c) Write the following product of cycles (in S_6) in 2-line notation: $(26)(12)(53)(34)(1264)$.
   (d) List all homomorphisms $\mathbb{Z}_6 \to S_3$.
   (e) List all homomorphisms $\mathbb{Z}_4 \to D_4$.
3. Prove that S_n is non-abelian for all $n \geq 3$.
4. Section 7 # 10
5. Section 8 # 4, 8, 35, 44, 49