Homework 6

Math 415 (section 502), Fall 2015

This homework is due on Thursday, October 8. You may cite results from class.

0. *(This problem is not to be turned in.)*
   (a) Read Section 10.
   (b) Section 10 # 20, 34, 40
   (c) Is 5 a generator of $\mathbb{Z}_6$? Is 5 a generator of $\mathbb{Z}_{10}$?
   (d) Let H be a subgroup of G. If $\phi : G \to G'$ is a homomorphism, is the restriction to H ($\phi|_H : H \to G'$) a homomorphism?
   (e) If $G \to G'$ is a homomorphism, and G is abelian, does it follow that G' is abelian?
1. Section 9 # 13, 23
2. Section 10 # 6, 19, 24
3. (No proofs necessary for this problem, but show your work.)
   (a) What is the order of an n-cycle? (A permutation is an *n-cycle* if it is a cycle of length n.)
   (b) What is the inverse of the permuation $(13)(256)$?
   (c) Is $(13)(245)(1689) \in S_9$ even or odd? What are its orbits?
   (d) Does S_3 have a cyclic subgroup of order 6? Does S_5?
4. The *kernel* of a homomorphism $\phi : G \to G'$ is the inverse image of the identity element in G', that is, $\ker(\phi) := \phi^{-1}[\{e_{G'}\}]$.
   (a) What operation makes $\{1, -1\}$ a group? (No proof necessary.)
   (b) Define the function

   $$\text{sign} : \ S_n \to \{1, -1\}$$

   by $\text{sign}(\sigma) := 1$ if σ is even, and $\text{sign}(\sigma) := -1$ if σ is odd. Is sign a homomorphism? Give a proof.
   (c) What is the kernel of sign? Give a proof.
5. Let H be a subgroup of a group G. Let $a, b \in G$. Prove that $aH = bH$ if and only if $a^{-1}b \in H$.

REMINDER: Exam 1 is on Thursday, October 8 (during class). The topics for the exam are from *Sections 0–10*, including: groups (such as $\mathbb{Z}$, $\mathbb{Z}_n$, $\mathbb{R}$, S_n, D_n, A_n), subgroups, abelian groups, cyclic groups (and their classification), homomorphisms (including $\mathbb{Z} \to G$ and $\mathbb{Z}_n \to G$) and isomorphisms, orders (of groups and elements), generating sets, permutations (including cycles and even vs. odd), Cayley's theorem, cosets, Lagrange's theorem.