Homework 6

Math 416 (sections 200 [Honors] and 500 [Regular]), Spring 2017

This homework is due on Thursday, February 23. *As a reminder, please write your section number on the top of your homework.*

0. *(This problem is not to be turned in.)*
   (a) Read Section 34
   (b) (Practice Problems) Section 34 #2, 9

1. An **automorphism** of a field F is a ring isomorphism $\phi : F \to F$.
   (a) Does complex conjugation ($a + bi \mapsto a - bi$) define an automorphism of $\mathbb{C}$? Prove your answer.
   (b) Complete the following sentence, and give a proof: *The set of all automorphisms of a field F forms a group in which the group operation is* ____________________.

2. Let F be a finite field of characteristic p. Consider the function:

$$\begin{aligned} \sigma : \ & F \to F \\ & x \mapsto x^p \ . \end{aligned}$$

   (a) Prove that σ is a ring homomorphism.
   (b) Prove that σ is one-to-one.
   (c) Conclude that σ is onto, and then that σ is an automorphism of F.

3. Section 34 #4

4. Consider the subgroups $H = \langle 25 \rangle$ and $K = \langle 50 \rangle$ of the group $G = \mathbb{Z}_{100}$.
   (a) List the cosets in G/H.
   (b) List the cosets in G/K.
   (c) List the cosets in H/K.
   (d) List the cosets in $(G/K)/(H/K)$.
   (e) Specify the bijection – between your answers to (a) and (d) – that comes from the Third Isomorphism Theorem.

5. (Honors only!) Are there countably or uncountably many finite fields contained in $\overline{\mathbb{Z}_p}$? Explain your answer.

6. (Honors only!) State and prove a ring-theory version of the (group theory) Third Isomorphism Theorem.

7. (Honors only!) Give an example of a group G and subgroups H and N for which $HN := \{hn \mid h \in H, \ n \in N\}$ is *not* a subgroup of G.