Homework 15 (the last homework[1])

Math 469 (section 500), Spring 2016

This homework is due on Thursday, April 28.

0. Read (or remind yourself) what *Descartes's rule of signs* says (page 24 of your textbook, or find it online).

1. Consider the polynomial $f(x) = x^3 + ax + b$, where $a, b \in \mathbb{R}$.

   (a) What is the maximum number of positive roots that f can have? Explain. (A *positive root* of f is some $x^* > 0$ for which $f(x^*) = 0$).

   (b) Give an example of $a, b \in \mathbb{R}$ so that f has *fewer* positive roots than the maximum number you found in (a). Justify your answer.

   (c) Give an example of $a, b \in \mathbb{R}$ so that f has exactly the maximum number of positive roots you found in (a). Justify your answer.

2. Consider the following iron-trafficking model, which may be viewed as a much-simplified version of the Chifman *et al.* (2012) model we looked at in class:

$$\begin{aligned}
\frac{dC}{dt} &= k_1 \left(\frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - k_2 C \left(1 - \frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - k_3 C - \alpha C \\
\frac{dF}{dt} &= k_2 C \left(1 - \frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - \alpha F \ ,
\end{aligned}$$

   where C represents iron in the cytosol and F represents ferritin, and n is a positive integer, $k_i > 0$ for all i, $\alpha > 0$, $T > 0$.

   (a) How does the number of steady states $(C, T) \in \mathbb{R}^2_{>0}$ depend on the values of n, k_i, T, α?

   (b) For each steady state, is it locally stable? Does the stability depend on n, k_i, T, α?

   (c) *(Optional bonus problem)* Is this system globally stable?

[1] we will continue to do in-class peer evaluations