Homework 4

Math 469, Spring 2024

This homework is due on Friday, February 9 at 11:30 am. (Turn in your answers – via Gradescope – to questions 1–3.)

0. *(This problem is not to be turned in.)* Read Sections 1.5–1.7.

1. Assume that the following second-order homogeneous linear difference equation has an **eigenvalue λ that is a double root**:

$$X_{t+2} + aX_{t+1} + bX_t = 0 . \tag{1}$$

(Assume that $b \neq 0$.) In class, we stated that two linearly independent solutions are $X(1)_t = \lambda^t$ and $X(2)_t = t\lambda^t$. The aim of this problem is to confirm that these are linearly independent and that $X(2)_t$ is indeed a solution.

   (a) Show that $X(1)_t$ and $X(2)_t$ are linearly independent.

   (b) Explain why $\lambda^2 = b$.

   (c) Explain why $a = -2\lambda$.

   (d) Use (b) and (c) to show that $X(2)_t$ satisfies the difference equation (1) for all $t = 0, 1, 2, \ldots$.

2. Section 1.8 #14, 17, 19, 22, 25 (*Hint:* Some of the matrices in #19 are not Leslie matrices, so please read the definitions of "irreducible" and "primitive" in the textbook.)

3. (In this part of your homework, you will start planning your final project.)

   (a) Who will be your partner for the final project? Or, you may do a solo project.

   (b) Together with your partner, pick 3 mathematical biology papers (for instance, from the list on the class `Piazza` site) that you might like to analyze for your project. State the titles and authors.

   In class on Friday, Feb. 9, please bring a laptop with these papers downloaded, or printed versions of the PDFs. You will pick a paper in class that day, with guidance from the instructor.