Homework 6

Math 469, Spring 2024

This homework is due on Friday, Feb. 23 at 11:30 am. (Turn in your answers – via Gradescope – to questions 1–5.)

0. Read Section 2.7

1. (a) *Complete the following sentence:*
   In class, we found that for ____ $< r <$ ____, the discrete logistic equation $x_{t+1} = rx_t(1 - x_t)$ has a 2-cycle $\{\bar{x}_1, \bar{x}_2\}$, where $\bar{x}_1 =$ _____ and $\bar{x}_2 =$ _____.

   (b) Show that this 2-cycle is locally asymptotically stable if $3 < r < 1 + \sqrt{6}$, and unstable when $r > 1 + \sqrt{6}$.

2. Let f be a function. How are the equilibria of $x_{t+1} = f^2(x_t)$ related to the equilibria and 2-cycles of $x_{t+1} = f(x_t)$? Explain. (*Note:* $f^2(x) = f(f(x))$.)

3. What is a *hyperbolic* equilibrium (see Definition 2.4)?

4. Section 2.12 #11–15

5. Section 2.12 #16–18 (analyze ONLY the difference equations that we did NOT reach in class).