Homework 1

Math 653, Fall 2019

This homework is due on Thursday, August 29.

1. Complete the survey (separate handout).
2. Read the Introduction of Hungerford, sections 1–4 and 6.
   (a) List all definitions you have *not* seen in a previous class.
   (b) List all partitions of $\{1, 2, 3\}$.
   (c) How many equivalence relations on $\{1, 2, 3\}$ are there? Explain.
3. Prove or disprove the following **subgroup criterion**: A subset H of a group G is a subgroup if and only if H is nonempty and for all $a, b \in H$, $ab^{-1} \in H$.
4. Let $f : G \to H$ be a group homomorphism. Prove or disprove the following:
   (a) $f(e_G) = e_H$.
   (b) $f(g^{-1}) = f(g)^{-1}$ for all $g \in G$.
   (c) $\mathrm{Ker}(f)$ is a subgroup of G.
   (d) $\mathrm{Im}(f)$ is a subgroup of H.
   (e) f is surjective.
5. Let G be a group such that $g^2 = e_G$ for all $g \in G$. Prove or disprove: G is abelian.
6. Let G be a group, and let $g \in G$. Prove or disprove:
   (a) The *center* of G – the set $C(G) := \{a \in G \mid ax = xa \text{ for all } x \in G\}$ – is a subgroup of G.
   (b) The *centralizer* of g – the set $C_G(g) := \{a \in G \mid ag = ga\}$ – is a subgroup of G.
7. (a) What do you think is the most important thing to gain in Math 653?
   (b) How do you plan to study for the quizzes and exams?