Homework 6

Math 653, Fall 2019

This homework is due on Thursday, October 3.

1. Read Hungerford, section 1.9
   (a) Section 1.8 #7, 12
   (b) Section 1.9 #5, 7, 11, 15
   (c) *(These problems are not to be turned in.)* Section 1.8 #3, 14
   (d) *(These problems are not to be turned in.)* Section 1.9 #1, 6, 8, 9, 10, 12
   (e) *(These problems are not to be turned in.)* Which of the following form the objects of a concrete category: finite groups, infinite groups, abelian groups? For which of these categories do the following exist: products, coporducts, free products?
   (f) *(This problem is not to be turned in.)* Is S_3 an internal direct sum of nontrivial subgroups?
2. Is S_4 the internal direct sum of nontrivial subgroups? Prove your answer.
3. *Prove or disprove*: The group $F_2 \times \mathbb{Z}$ is isomorphic to $F_2 \times F_2$, where F_2 denotes the free group on a two-element set.
4. *Prove or disprove*: Let n be a fixed integer. Let F be a free group (on some set), and let N be the subgroup generated by the set $\{x^n \mid x \in F\}$. Then N is normal in F.