Homework 4

Math 669, Spring 2022

This homework is due on Monday, February 28

1. From *Mathematical Biology* (Murray 2002), Chapter 2 (page 76), #2 and 5

2. (a) Give an example of a difference equation $N_{t+1} = f(N_t)$ with three steady states – one locally asymptotically stable and two unstable. (Explain your answer.)

   (b) Give an example of an ODE $\frac{dN}{dt} = f(N)$ with three steady states – one locally asymptotically stable and two unstable. (Explain your answer.)

3. Determine the limiting behavior of solutions to the difference equation

$$N_{t+1} \quad = \quad aN_t + b \ ,$$

   where $a, b \in \mathbb{R}$. (How) does your answer depend on a, b, and N_0 (the initial value)?

4. Consider the population model $N_{t+1} = f(N_t)$, where

$$f(x) \ = \ x \exp\left(r\left(\frac{K-x}{K}\right)\right) \ ,$$

   with $r > 0$ and $K > 0$.

   (a) Determine the steady states and their stability. (How) do your answers depend on r and/or K?

   (b) Show that (for every $K > 0$) there is a 2-cycle when $r > 0$.

   (c) Are there any 2-cycles when $r \leq 0$? Explain.

5. Consider the following predator-prey model (where $b > 0$):

$$\begin{aligned} \frac{dx}{dt} &= x(1-x-y) \\ \frac{dy}{dt} &= y(-b+x) \ . \end{aligned}$$

   (a) Which species variable represents the predator (and which represents the prey)? Explain.

   (b) Find the steady states, and determine their stability. (How) do your answers depend on b?