Spring2015 Math 151

Sample Questions for Exam 3

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Review Exercises: Sections 4.3 - 6.1

Section 4.3

1. Evaluate $\log_3 108 - \log_3 4$

2. Solve for x: $\log(x+3) + \log(x) = 1$

3. Solve for x: $\ln x - \ln(x+1) = \ln 2 + \ln 3$

4. Find $\lim_{x \to 2^+} \ln(x-2)$

5. Find $\lim_{x \to \infty} [\log(2x^2 - 1) - \log(3x^2 + 6)]$

6. What is the domain of $f(x) = \ln(x^2 + 2x - 8)$?

Section 4.4

7. Find $f'(x)$ for $f(x) = \ln(2x^2 - 8)$

8. Find the derivative of $f(x) = 2^{\cos x} + \log_7(3x - 1)$

9. Find y' for $y = (\cos x)^{\tan x}$

10. Find the slope of the tangent line to the curve $f(x) = x\ln(x)$ at $x = e^2$.

Section 4.5

11. At a certain instant, 100 grams of a radioactive substance is present. After 4 years, 20 grams remain.

    a.) What is the half life of the substance?

    b.) How much of the substance remains after 2.5 years?

12. A bowl of soup at temperature 180° is placed in a 70° room. If the temperature of the soup is 150° after 2 minutes, when will the soup be an eatable 100°?

Section 4.6

13. Express $\tan(\arcsin x)$ as an algebraic expression.

14. Find the derivative of $y = x^2 \arccos(e^{3x})$

15. Find the equation of the line tangent to $y = \tan^{-1}(2x - 1)$ when $x = 1$.

16. Compute the exact value of $\lim_{x \to \infty} \arccos\left(\frac{1+2x}{5-4x}\right)$

17. Compute $\sec(\arctan(-\sqrt{5}))$

18. Compute $\arcsin(\sin \frac{4\pi}{3})$

Section 4.8

19. Find the limits of each of the following:

    a) $\lim_{x \to 0} \frac{\arcsin(3x)}{2x}$

    b) $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$

    c) $\lim_{x \to 0^+} \frac{\ln x}{\sqrt{x}}$

    d.) $\lim_{x \to \pi/2^-} (\sec x - \tan x)$

    e.) $\lim_{x \to 1^+} (x-1)\tan(\pi x/2)$

    f.) $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^{4x}$

Section 5.1 - 5.3

20. If $f(x) = \frac{1}{x}$, verify $f(x)$ satisfies the Mean Value Theorem on the interval $[1, 10]$ and find all c that satisfies the conclusion of the Mean Value Theorem.

21. Find the absolute maximum and minimum of the given function on the given interval.

    a) $x^3 - 5x^2 + 3$ on $[-1, 3]$

    b) $x \ln x$ on $[1, e]$

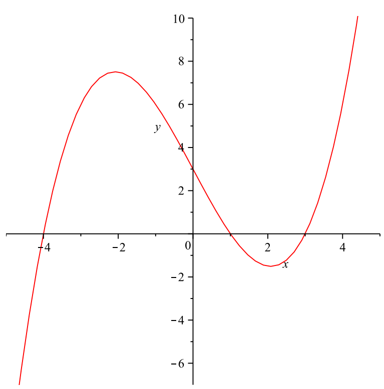
22. Find the intervals where the given function is increasing and decreasing, local extrema, intervals of concavity and inflection points.

    a) $f(x) = x^3 - 2x^2 + x$

    b) $f(x) = xe^{2x}$

23. Find the concavity of f if $f'(x) = \frac{\ln x}{x}$

24. In the graph that follows, the graph of f' is given. Using the graph of f', determine all critical values of f, where f is increasing and decreasing, local extrema of f, where f is concave up and concave down, and the x-coordinates of the inflection points of f. Assume f is continuous.



Section 5.5

25. A cardboard rectangular box holding 32 cubic inches with a square base and open top is to be constructed. If the material for the base costs \$2 per square inch and material for the sides costs \$5 per square inch, find the dimensions of the cheapest such box.

26. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of length 3 m and 4 m if two sides of the rectangle lie along the legs.

Section 5.7

27. Find an antiderivative of $\frac{1}{\sqrt{1-x^2}} - \frac{1+x}{x}$.

28. Given $f''(x) = 2e^x - 4\sin(x)$, $f(0) = 1$, and $f'(0) = 2$, find $f(x)$.

29. Find the vector functions that describe the velocity and position of a particle that has an acceleration of $\mathbf{a}(t) = \langle \sin t, 2 \rangle$, initial velocity of $\mathbf{v}(0) = \langle 1, -1 \rangle$ and an initial position of $\mathbf{r}(0) = \langle 0, 0 \rangle$.

Section 6.1

30. $\sum_{i=2}^{5} i^2 =$

31. Write $1 + \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \frac{1}{e^5}$ in summation notation.

32. $\sum_{i=3}^{99} \left(\frac{1}{i} - \frac{1}{i+1} \right) =$