Spring 2015 Math 151

Week in Review 3

courtesy: Amy Austin

(covering Sections 2.3, 2.5 and 2.6)

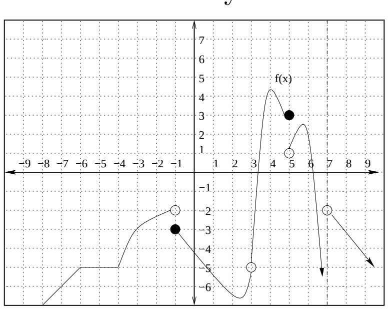
Section 2.3

Compute the exact value of the following limits. If the limit does not exist, support your answer by evaluating left and right hand limits.

1. $\lim_{x\to 1}(4x^3 - 3x + 1)$
2. $\lim_{x\to -5} \dfrac{x^2+5x}{x+5}$
3. $\lim_{x\to 2} \dfrac{x-\sqrt{3x-2}}{x^2-4}$
4. $\lim_{h\to 0} \dfrac{(3+h)^{-1}-3^{-1}}{h}$
5. $\lim_{x\to 1} \dfrac{x-4}{x-1}$
6. $\lim_{x\to 3} f(x)$, where $f(x) = \begin{cases} x+5 & \text{if } x \le 3 \\ x^3-3 & \text{if } x > 3 \end{cases}$
7. $\lim_{x\to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$
8. $\lim_{x\to 2} \dfrac{x^2-4}{|\, x-2 \,|}$
9. $\lim_{x\to 1} f(x)$ if it is known that $4x \le f(x) \le x+3$ for all x in $[0,2]$.

Section 2.5

10. Referring to the graph, explain why the function $f(x)$ is or is not continuous (you decide which) at $x=-1$, $x=3$, $x=5$, $x=-4$ and $x=7$. For the values of x where $f(x)$ is not continuous, is it continuous from the right, left or neither? In addition, for each discontinuity, is it a jump discontinuity, infinite discontinuity or a removable discontinuity?



11. Sketch the graph of $f(x)$ and determine where the function

$$f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ 4x & -1 \le x < 1 \\ 3 & x = 1 \\ 5-x & \text{if } x > 1 \end{cases}$$

is not continuous. Fully support your answer.

12. Which of the following functions has removable discontinuity at $x=a$? If the discontinuity is removable, find a function g that agrees with f for $x \ne a$ and is continuous at $x=a$. Note: f has removable discontinuity at $x=a$ if $\lim_{x\to a} f(x)$ exists and $f(x)$ can be redefined so that $\lim_{x\to a} f(x) = f(a)$ (thereby removing the discontinuity).

(a) $f(x) = \dfrac{x^2-4}{x-2}$, $x=2$.

(b) $f(x) = \dfrac{1}{x-1}$, $x=1$

(c) $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x+4 & \text{if } x \ge 1 \end{cases}$, $x=1$

13. If $f(x) = \dfrac{x+2}{x^2+5x+6}$, find all values of $x=a$ where the function is discontinuous. For each discontinuity, find the limit as x approaches a, if the limit exists. If the limit does not exist, support your answer by evaluating left and right hand limits.

14. Suppose it is known that $f(x)$ is a continuous function defined on the interval $[1,5]$. Suppose further it is given that $f(1)=-3$ and $f(5)=6$. Give a graphical arguement that there is at least one solution to the equation $f(x)=1$.

15. If $g(x) = x^5 - 2x^3 + x^2 + 2$, use the Intermediate Value Theorem to find an interval which contains a root of $g(x)$, that is contains a solution to the equation $g(x)=0$.

16. Find the values of c and d that will make

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2+d & \text{if } 1 \le x \le 2 \\ 4x & \text{if } x > 2 \end{cases}$$

continuous on all real numbers. Once the value of c and d is found, find $\lim_{x\to 1} f(x)$ and $\lim_{x\to 2} f(x)$.

Section 2.6

17. Compute the following limits:

a.) $\lim_{x\to\infty} \dfrac{4x^3 - 6x^4}{2x^3 - 9x + 1}$

b.) $\lim_{t\to-\infty} \dfrac{t^9 - 4t^{10}}{t^{12} + 2t^2 + 1}$

c.) $\lim_{x\to\infty} \dfrac{4x - 6x^3}{-2x^3 - 9x + 1}$

d.) $\lim_{x\to\infty} \dfrac{\sqrt{2 + x^2}}{4 - 7x}$

e.) $\lim_{x\to-\infty} \dfrac{\sqrt{5x^2 + 1}}{x - 3}$

f.) $\lim_{x\to\infty} (\sqrt{x^2 + 5x + 1} - x)$

g.) $\lim_{x\to-\infty} (x + \sqrt{x^2 + x + 2})$

18. Find all horizontal and vertical asymptotes of $f(x) = \dfrac{x^3}{x^3 - x}$