Spring 2015 Math 151

Week in Review 5

courtesy: Amy Austin

(Covering 3.5-3.8)

Section 3.5

1. Find the derivative of the following functions:

   a.) $f(x) = (x^3 + x + 1)^8$

   b.) $f(x) = \sqrt{x^5 - \dfrac{3}{x^2} + \sin(x) - \sec(x)}$

   c.) $f(x) = \dfrac{1}{(x^2 + x - 1)^2}$

   d.) $h(x) = \tan(x^2)$

   e.) $g(x) = \cos^3(x^2 + a^2)$

   f.) $g(x) = \sin^3(x^2) + \cot(\sin(2x))$

   g.) $f(x) = (2x + 1)^5(\sqrt{x} - x + 3)^7$

   h.) $h(x) = \dfrac{x}{(x^5 + 1)^4}$

2. Given $h = f \circ g$, $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, $f'(6) = 7$. Find $h'(3)$.

3. Suppose that $F(x) = f(x^4)$ and $G(x) = (f(x))^4$. Also, suppose it is given that $f(2) = -1$, $f(16) = 3$, $f'(2) = -2$ and $f'(16) = 4$. Compute $F'(2)$ and $G'(2)$.

4. If $G(t) = (t + f(\tan 2t))^3$, find an expression for $G'(t)$.

Section 3.6

5. Find $\dfrac{dy}{dx}$ if $x^4 - 4x^2y^2 + y^3 = 0$

6. Find $\dfrac{dy}{dx}$ for $\cos(2x) - \sin(x + y) = 1$

7. Find the equation of the line tangent to $x^2 + y^2 = 2$ at (1,1).

8. Regard y as the independent variable and x as the dependent variable, and use implicit differentiation to find $\dfrac{dx}{dy}$ for the equation $(x^2 + y^2)^2 = 2x^2y$.

Section 3.7

9. Find the angle between the tangent vector and the position vector for $\mathbf{r(t)} = \langle t^2, 2t^3 \rangle$ at the point where $t = -1$.

10. Find the vector and parametric equations of the line tangent to $\mathbf{r(t)} = \langle t^3 + 2t, 4t - 5 \rangle$ at the point where $t = 2$.

11. Sketch the curve $\mathbf{r(t)} = \langle t^2, t \rangle$. Find the tangent and unit tangent vector to the curve at the point $(4, 2)$. Draw the position and tangent vector along with the sketch of the curve at the point $(4, 2)$.

12. Find the angle of intersection of the curves

    $\mathbf{r_1(s)} = \langle s - 2, s^2 \rangle$ and $\mathbf{r_2(t)} = \langle 1 - t, 3 + t^2 \rangle$

Section 3.8

13. Find y'' for $y = \sqrt{x^2 + 1}$.

14. If $\mathbf{r(t)} = \langle t^3, t^2 \rangle$ represents the position of a particle at time t, find the angle between the velocity and the acceleration vector at time $t = 1$.

15. Find the $98th$ derivative of:

    a.) $f(x) = \dfrac{1}{x^2}$

    b.) $f(x) = \sin(3x)$