

Spring 2015 Math 151

Week in Review 6

courtesy: Amy Austin

(Covering 3.9-3.11)

Section 3.9

1. Given $x = \cos t$ and $y = t^2$, find $\frac{dy}{dx}$. Next, find the equation of the tangent line at $t = \frac{\pi}{4}$.
2. Let $x = t^4 - 4t^3$ and $y = 3t^2 - 6t$.
 - a.) Find the equation of the tangent line at the point (5, 9).
 - b.) Find all point(s) on the curve where the tangent line is vertical or horizontal.
3. Show the curve $x = \cos t$ and $y = \sin t \cos t$ has two tangents at (0, 0). Find the equations of these tangent lines.
4. At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent line parallel to the line with equations $x = -7t$, $y = 12t - 5$?

Section 3.10

5. Water leaking onto a floor creates a circular pool with an area that increases at a rate of 3 square inches per minute. How fast is the radius of the pool increasing when the radius is 10 inches?
6. When a rocket is 2 miles high, it is moving vertically upward at a speed of 300 mph. At that instant, how fast is the angle of elevation of the rocket increasing, as seen by an observer on the ground 5 miles from the launching pad?
7. A filter in the shape of a cone is 6 inches high and has a radius of 2 inches at the top. A solution is poured in the cone so that the water level is rising at a rate of $\frac{3}{2}$ inches per second. How fast is the water being poured in when the water level has a depth of 2 inches?.

8. One end of a 13 foot ladder is on the ground, and the other end rests on a vertical wall. If the top of the ladder is being pushed up the wall at a rate of 1 foot per second, how fast is the base of the ladder approaching the wall when it is 3 feet from the wall?
9. A point moves around the circle $x^2 + y^2 = 9$. When the point is at $(-\sqrt{3}, \sqrt{6})$, its x coordinate is increasing at a rate of 20 units per second. How fast is its y coordinate changing at that instant?
10. Two sides of a triangle have lengths 5 m and 4 m. The angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is 60° .
11. A trough is 10 feet long and its ends have the shape of isosceles triangles that are 3 feet across the top and have a height of 1 foot. If the trough is filled with water at a rate of 12 cubic feet per minute, how fast is the water level rising when the water is 6 inches deep?

Section 3.11

12. Let $y = 4 - x^2$. Find Δy if x changes from $x = 1$ to $x = 1.5$.
13. If $f(x) = 4 - x^2$, find dy if $x = 1$ and $dx = \frac{1}{2}$.
14. Find the differential dy if $y = \frac{r}{r+1}$ and $dr = 0.5$.
15. Use differentials to approximate $(1.97)^6$.
16. Use differentials to approximate $\cos(31.5^\circ)$
17. Find the linear approximation for $f(x) = \frac{1}{x}$ at $x = 4$.
18. Find the linear approximation for $f(x) = \sqrt[3]{x+1}$ at $x = 0$ and use it to approximate $\sqrt[3]{0.95}$

19. The radius of a circular disk is given to be 24 cm with a maximum error in measurement of 0.2 cm.
- (a) What is the maximum error in the calculated area of the disk?
 - (b) Use differentials to approximate the maximum error in the area of the disk.
 - (c) What is the relative error?
20. Suppose for a function f , the linear approximation for $f(x)$ at $a = 3$ is given by $y = 2x + 7$.
- a.) Find the value of $f'(3)$ and $f(3)$.
 - b.) If $g(x) = \sqrt{f(x)}$, find the linear approximation for $g(x)$ at $a = 3$.
21. Find the quadratic approximation for $f(x) = \cos x$ at $x = \frac{\pi}{3}$.