1. What is the slope of the line with parametric equations $x = 2t + 3$, $y = 7t - 2$?

a) $\frac{7}{2}$

b) $\frac{-2}{7}$

c) $\frac{3}{2}$

d) $\frac{-2}{3}$

e) $\frac{2}{7}$

$y = mx + b$

$\vec{v} = \langle 2, 7 \rangle$

$m = \frac{7}{2}$

2. What value of x makes the vectors $\langle 1, x \rangle$ and $\langle 3 - 4x, 5 \rangle$ perpendicular?

a) $x = -3$

b) $x = -1$

c) $x = 0$

d) $x = 1$

e) $x = 3$

$\vec{a} \perp \vec{b}$ if $\vec{a} \cdot \vec{b} = 0$

$\langle 1, x \rangle \cdot \langle 3 - 4x, 5 \rangle = 0$

$1(3 - 4x) + x(5) = 0$

$x + 3 = 0$

$\boxed{x = -3}$

3. Which of the following gives parametric equations of the line passing through $(-1, 1)$ and perpendicular to the line $\boxed{x = 4 - 3t,\ y = 5 + t?}$

$\vec{r}_0$

$\vec{v} = \langle -3, 1 \rangle$

$v_1^{\perp} = \langle 1, 3 \rangle$

$v_2^{\perp} = \langle -1, -3 \rangle$

a) $\mathbf{r}(t) = \langle -2 - t, -2 + t \rangle$

b) $\mathbf{r}(t) = \langle -1 - t, 1 - 3t \rangle$

c) $\mathbf{r}(t) = \langle -1 + t, 1 + 3t \rangle$

d) $\mathbf{r}(t) = \langle -2 - 3t, -2 - t \rangle$

e) both (b) and (c) are correct

Answer!

Vector equation of the line is:

① $\vec{r}_0 + t v_1^{\perp}$

$\langle -1, 1 \rangle + t \langle 1, 3 \rangle$

$x = -1 + t$

$y = 1 + 3t$

② $r_0 + t v_2^{\perp}$

$\langle -1, 1 \rangle + t \langle -1, -3 \rangle$

$x = -1 - t$

$y = 1 - 3t$

4. $\lim_{x \to 4} \dfrac{2x^2 - 32}{x - 4} =$

a) 1

b) 0

c) 2

d) does not exist

e) 16

$$\lim_{x \to 4} \frac{2(x^2 - 16)}{x - 4} = \lim_{x \to 4} \frac{2(x+4)\cancel{(x-4)}}{\cancel{x-4}} = 2(4+4)$$

$$= \boxed{16}$$

5. $\lim_{x\to 0+} \frac{x^2 - 2x}{x} =$

a) 0 b) 2 c) $-\infty$

d) ∞ e) -2

$$\lim_{x\to 0^+} \frac{\cancel{x}(x-2)}{\cancel{x}} = \boxed{-2}$$

6. $\lim_{x\to 0^-} \frac{x^2 - 2x}{|x|} =$

a) 0 b) 2 c) $-\infty$

d) ∞ e) 1

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\lim_{x\to 0^-} \frac{x(x-2)}{|x|} = \lim_{x\to 0^-} \frac{\cancel{x}(x-2)}{-\cancel{x}} = \frac{-2}{-1} = \boxed{2}$$

7. $\lim_{x \to 0^+} \frac{x-2}{x} =$ $\frac{-2}{0}$

a) 0 b) -2 c) $-\infty$

d) ∞ e) 1

$\lim_{x \to 0^+} \frac{x-2}{x}$ $\frac{-}{+} = \boxed{-\infty}$

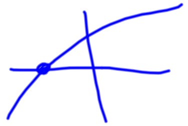
8. According to the Intermediate Value Theorem, the equation $x^3 - 2x^2 + x = -5$ has a solution in which of the following intervals?

~~a)~~ $[-3, -2]$ b) $[2, 3]$

c) $[-2, -1]$ d) $[-1, 0]$

e) $[0, 1]$

$x^3 - 2x^2 + x + 5 = 0$



which interval contains a solution to this equation?

$f(x) = x^3 - 2x^2 + x + 5$

a) $f(-3) = -27 - 18 - 3 + 5 < 0$

$\boxed{f(-2) = -8 - 8 - 2 + 5 < 0}$

c) $\boxed{f(-1) = -1 - 2 - 1 + 5 > 0}$

$f(-2) < 0$

$f(-1) > 0$

solution to $f(x) = 0$ exists on $[-2, -1]$.

9. $\lim_{x \to 1} \frac{x+1}{(x-1)^2} =$ $\frac{2}{0}$

a) 0 b) does not exist c) $-\infty$

d) ∞ e) 1

$\lim_{x \to 1^+} \frac{x+1}{(x-1)^2} = \infty$

$\lim_{x \to 1^-} \frac{x+1}{(x-1)^2} = \infty$

10. If $f(x) = \begin{cases} 5 - \frac{2}{5}x & \text{if } x < 5 \\ 3 & \text{if } 5 < x < 8 \\ 9 - x & \text{if } x > 8 \end{cases}$, determine which of the following statements is true.

~~a)~~ f is continuous at $x = 5$

~~b)~~ $\lim_{x \to 5} f(x)$ does not exist.

~~c)~~ $\lim_{x \to 8+} f(x) = 3 \leftarrow$ $\lim_{x \to 8^+} f(x) = \lim_{x \to 8^+} (9 - x) = 1$

d) $\lim_{x \to 5} f(x) = 3$

~~e)~~ f is continuous for all values of x.

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (3) = 3$$

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} \left(5 - \frac{2}{5}x\right) = 3$$

11. Find the work done by a force of 20 Newtons acting in the direction $N25°W$ in moving an object 4 meters due west.

a) $20\cos(25°)$ Joules

b) $80\cos(25°)$ Joules

c) $80\cos(65°)$ Joules

d) $20\cos(65°)$ Joules

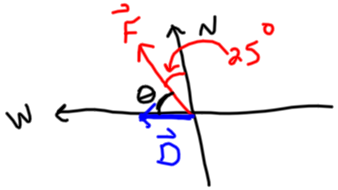
e) None of the avove

① $W = \vec{F} \cdot \vec{D}$

$\vec{F}$ = force vector

$\vec{D}$ = displacement vector

② $W = |\vec{F}||\vec{D}|\cos\theta$, θ = angle between $\vec{F}$ and $\vec{D}$



$|\vec{F}| = 20\,N$ $\quad \theta = 90° - 25°$

$|\vec{D}| = 4\,m$ $\quad \theta = 65°$

$$W = (20N)(4m)\cos(65°) = \boxed{80\cos(65°)\,J}$$

12. Given the points $P(4,-4)$ and $Q(5,-2)$, find a unit vector in the direction of the vector starting at P and ending at Q.

a) $\frac{1}{\sqrt{5}}\mathbf{i}+\frac{2}{\sqrt{5}}\mathbf{j}$ b) $\frac{140}{\sqrt{29}}\mathbf{i}-\frac{56}{\sqrt{29}}\mathbf{j}$ c) $9\mathbf{i}-6\mathbf{j}$

d) $\mathbf{i}+2\mathbf{j}$ e) $\frac{9}{\sqrt{117}}\mathbf{i}-\frac{6}{\sqrt{117}}\mathbf{j}$

$$\vec{PQ} = \langle 1, 2 \rangle$$

$$\vec{u} = \frac{\langle 1, 2\rangle}{|\langle 1,2\rangle|} = \frac{\langle 1,2\rangle}{\sqrt{5}} = \boxed{\frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}}$$

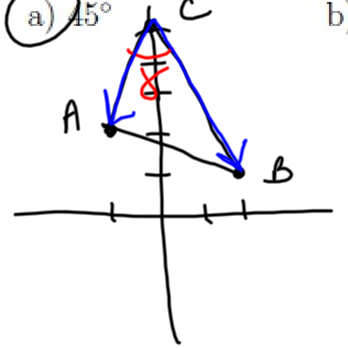
13. $\lim\limits_{x\to\infty} \frac{6x^2-x-3}{2+3x-3x^2} =$

a) 3 b) 1 c) ∞ d) -2 e) $-\frac{3}{2}$

$$\lim_{x\to\infty} \frac{\cancel{x^2}\left(6-\frac{1}{\cancel{x}}-\frac{3}{\cancel{x^2}}\right)}{\cancel{x^2}\left(\frac{2}{\cancel{x^2}}+\frac{3}{\cancel{x}}-3\right)} = \frac{6}{-3} = -2$$

14. The points $A(-1,2)$, $B(2,1)$, and $C(0,5)$ form a triangle. Find angle C.

a) 45°   b) 30°   c) 135°   d) 150°   e) 90°



$\vec{CA} = \langle -1, -3 \rangle$

$\vec{CB} = \langle 2, -4 \rangle$

$$\cos\gamma = \frac{\langle -1,-3\rangle \cdot \langle 2,-4\rangle}{|\langle -1,-3\rangle|\,|\langle 2,-4\rangle|}$$

$$\cos\gamma = \frac{-2+12}{\sqrt{10}\,\sqrt{20}} = \frac{10}{\sqrt{10}\sqrt{10\cdot 2}}$$

$$\cos\gamma = \frac{\cancel{10}}{\cancel{\sqrt{10}}\cancel{\sqrt{10}}\sqrt{2}}$$

$$\cos\gamma = \frac{1}{\sqrt{2}}$$

$$\boxed{\gamma = 45°}$$

15. The parametric curve determined by the equations $x = \sin t$, $y = \cos^2 t$, $0 \le t \le \frac{\pi}{2}$ forms:

$x^2 = \sin^2 t$

a) part of a parabola

b) part of a hyperbola

c) part of a circle

d) line segment

e) none of the above

$$\underbrace{\cos^2 t}_{y} + \underbrace{\sin^2 t}_{x^2} = 1 \longrightarrow y + x^2 = 1$$

$$y = 1 - x^2$$

16. Find all vertical asymptotes for the curve $\dfrac{x-2}{x^2-4}$

a) $x = 0$

b) $x = -2$ and $x = 2$

c) $x = -2$ only

d) $x = 2$ only

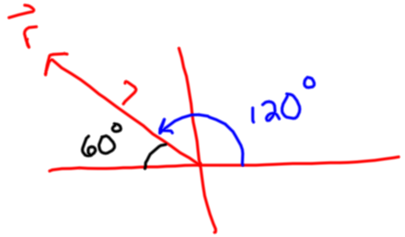
e) There are no vertical asymptotes.

Factor first, then set denominator = 0

$$\frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}}$$

vertical asymptote: $x=-2$

17. Find the components of the vector **r** given that the magnitude of **r** is 7 and **r** creates an angle of 120° with the positive x axis.



a) $\mathbf{r} = \left\langle \frac{7}{2}, \frac{7\sqrt{3}}{2} \right\rangle$

b) $\mathbf{r} = \left\langle \frac{7\sqrt{3}}{2}, \frac{7}{2} \right\rangle$

c) $\mathbf{r} = \left\langle -\frac{7\sqrt{3}}{2}, \frac{7}{2} \right\rangle$

d) $\mathbf{r} = \left\langle -\frac{7}{2}, -\frac{7\sqrt{3}}{2} \right\rangle$

e) $\mathbf{r} = \left\langle -\frac{7}{2}, \frac{7\sqrt{3}}{2} \right\rangle$

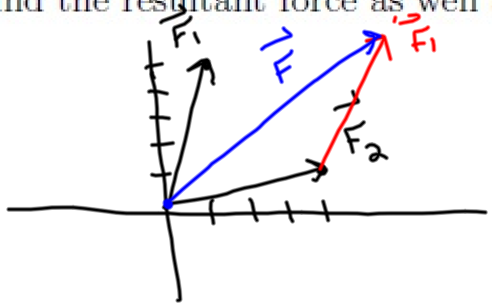
$\vec{r} = \langle -7\cos 60°, 7\sin 60° \rangle$

$\vec{r} = \langle -7\cdot\frac{1}{2}, 7\cdot\frac{\sqrt{3}}{2} \rangle$

Part II - Work Out Problems

All answers must be algebraically supported to receive full credit.

18. If two forces given by $\mathbf{F_1} = \langle 1, 5\rangle$ and $\mathbf{F_2} = \langle 4, 1\rangle$ are acting on an object sitting at the origin, find the resultant force as well as its magnitude and direction.



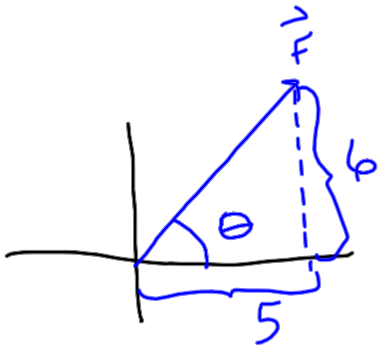
resultant force is

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$= \langle 1,5\rangle + \langle 4,1\rangle$$

Force $\longrightarrow$ $\vec{F} = \langle 5, 6\rangle$

magnitude $\longrightarrow$ $|\vec{F}| = \sqrt{25+36} = \sqrt{61}$



direction: $\tan\theta = \frac{6}{5}$

$$\boxed{\theta = \arctan\left(\frac{6}{5}\right)}$$

19. Use the limit definition to find the derivative, $f'(x)$, of $f(x) = \sqrt{2-3x}$. Next, find the slope of the tangent line to the graph of $f(x)$ at $x = -1$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2-3(x+h)} - \sqrt{2-3x}}{h} \cdot \frac{\sqrt{2-3(x+h)} + \sqrt{2-3x}}{\sqrt{2-3(x+h)} + \sqrt{2-3x}}$$

$$= \lim_{h \to 0} \frac{2-3(x+h) - (2-3x)}{h\left(\sqrt{2-3(x+h)} + \sqrt{2-3x}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{2} - \cancel{3x} - 3h - \cancel{2} + \cancel{3x}}{h\left(\sqrt{2-3(x+h)} + \sqrt{2-3x}\right)}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}\left(\sqrt{2-3(x+\underset{\downarrow 0}{h})} + \sqrt{2-3x}\right)}$$

$$= \frac{-3}{\sqrt{2-3x} + \sqrt{2-3x}} = \boxed{\frac{-3}{2\sqrt{2-3x}}}$$

so $f'(x) = \dfrac{-3}{2\sqrt{2-3x}}$

$m = f'(-1)$

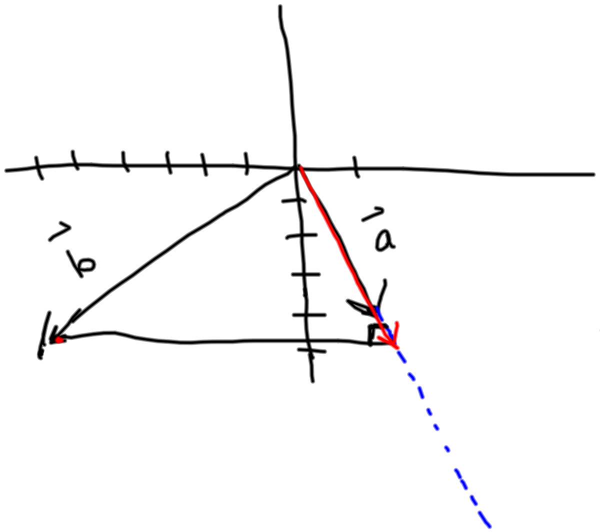
$$\boxed{m = \frac{-3}{2\sqrt{5}}}$$

equation:

point is $(-1, \sqrt{5})$

$$\boxed{\text{equation} \quad y - \sqrt{5} = \frac{-3}{2\sqrt{5}}(x+1)}$$

20. Find the vector projection and the scalar projection of $\langle -6, -5\rangle$ onto $\langle 1, -4\rangle$.



$\vec{b} = \langle -6, -5 \rangle$

$\vec{a} = \langle 1, -4 \rangle$

Scalar projection of $\vec{b}$ onto $\vec{a}$ is

$$\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{\langle 1, -4 \rangle \cdot \langle -6, -5 \rangle}{|\langle 1, -4 \rangle|}$$

$$= \frac{-6 + 20}{\sqrt{17}}$$

$$\boxed{\text{comp}_{\vec{a}} \vec{b} = \frac{14}{\sqrt{17}}}$$

Vector projection of $\vec{b}$ onto $\vec{a}$ is

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \frac{\vec{a}}{|a|}$$

$$= \frac{14}{\sqrt{17}} \cdot \frac{\langle 1, -4 \rangle}{\sqrt{17}} = \boxed{\frac{14}{17} \langle 1, -4 \rangle}$$

21. Find the distance from the point $\underbrace{(2,3)}_{P}$ to the line $y = 4x + 5$

Step1: Find any two points on the line $y=4x+5$

$A(0,5)$

$B(-1,1)$

Step 2: define $\vec{a} = \overrightarrow{AB}$

$\boxed{\vec{a} = \langle -1,-4 \rangle} \longrightarrow a^{\perp} = \langle 4,-1 \rangle$

$\vec{b} = \overrightarrow{AP}$, where $P = (2,3)$

$\boxed{\vec{b} = \langle 2,-2 \rangle}$

Step3: $d = \left| \text{comp}_{a^{\perp}} \vec{b} \right| = \dfrac{a^{\perp} \cdot \vec{b}}{|a^{\perp}|} = \dfrac{\langle 4,-1 \rangle \cdot \langle 2,-2 \rangle}{|\langle 4,-1 \rangle|}$

$= \dfrac{8+2}{\sqrt{17}} = \dfrac{10}{\sqrt{17}}$

22. Evaluate $\lim_{x \to 3} \frac{|x-3|}{x^2-9}$, if it exists. If the limit does not exist, support your answer by evaluating left and right hand limits.

$|x-3|$

$$\lim_{x \to 3} \frac{|x-3|}{(x+3)(x-3)}$$

$$|x-3| = \begin{cases} x-3, & x \geqslant 3 \\ -(x-3) & x < 3 \end{cases}$$

$$\lim_{x \to 3^+} \frac{|x-3|}{(x+3)(x-3)} = \lim_{x \to 3^+} \frac{\cancel{x-3}}{(x+3)\cancel{(x-3)}} = \boxed{\frac{1}{6}}$$

$$\lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$$

so $\lim_{x \to 3} f(x)$ dNE

$$\lim_{x \to 3^-} \frac{|x-3|}{(x+3)(x-3)} = \lim_{x \to 3^-} \frac{-\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} = \boxed{-\frac{1}{6}}$$

23. Find values of a and b which make $f(x)$ continuous for all x, if possible. If not possible, explain why.

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \leftarrow \\ ax^2 - bx + 3 & \text{if } 1 \le x < 2 \leftarrow \\ 2x - a + b & \text{if } x \ge 2 \end{cases}$$

① since each piece is continuous on it's doman

② $x=1$ + $x=2$ are in the doman

need to make $\lim\limits_{x\to 1} f(x)$ + $\lim\limits_{x\to 2} f(x)$ exist

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x^2-1}{x-1} = \lim_{x\to 1^-} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = 2$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (ax^2 - bx + 3) = a - b + 3$$

equation 1: $\boxed{2 = a - b + 3}$

23. Find values of a and b which make $f(x)$ continuous for all x, if possible. If not possible, explain why.

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \\ ax^2 - bx + 3 & \text{if } 1 \le x < 2 \\ 2x - a + b & \text{if } x \ge 2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax^2 - bx + 3) = 4a - 2b + 3$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (2x - a + b) = 4 - a + b$$

equation 2:
$4a - 2b + 3 = 4 - a + b$

equation 1:
$\boxed{2 = a - b + 3} \rightarrow \boxed{a = b - 1}$

so $4(b-1) - 2b + 3 = 4 - (b-1) + b$

$4b - 4 - 2b + 3 = 4 - \cancel{b} + 1 + \cancel{b}$

$2b = 6 \rightarrow \boxed{b = 3}$ $\qquad \boxed{a = 2}$

24. Find $\lim_{x\to 3} \dfrac{\frac{1}{x+4} - \frac{1}{7}}{x-3}$ $= \dfrac{\frac{1}{7} - \frac{1}{7}}{3-3} = \dfrac{0}{0} \rightarrow$ algebra!

$$\lim_{x\to 3} \frac{\frac{1}{x+4} - \frac{1}{7}}{x-3} \cdot \frac{(x+4)(7)}{(x+4)(7)}$$

$$\lim_{x\to 3} \frac{\frac{1}{\cancel{x+4}}\cancel{(x+4)}(7) - \frac{1}{\cancel{7}}(x+4)\cancel{(7)}}{(x-3)(x+4)(7)}$$

$$\lim_{x\to 3} \frac{7-(x+4)}{(x-3)(x+4)(7)} = \lim_{x\to 3} \frac{-x+3}{(x-3)(x+4)(7)}$$

$$= \lim_{x\to 3} \frac{-\cancel{(x-3)}}{\cancel{(x-3)}(x+4)(7)} = \frac{-1}{(7)(7)}$$

$$= \boxed{-\frac{1}{49}}$$

25. Find $\lim_{x\to\infty} \frac{\sqrt{10x^2-5}}{2-3x}$ and $\lim_{x\to-\infty} \frac{\sqrt{10x^2-5}}{2-3x}$

$$\sqrt{x^2} = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\lim_{x\to\infty} \frac{\sqrt{x^2\left(10-\frac{5}{x^2}\right)}}{x\left(\frac{2}{x}-3\right)} = \lim_{x\to\infty} \frac{\sqrt{x^2}\sqrt{10-\frac{5}{x^2}}}{x\left(\frac{2}{x}-3\right)}$$

$$= \lim_{\boxed{x\to\infty}} \frac{|x|\sqrt{10-\frac{5}{x^2}}}{x\left(\frac{2}{x}-3\right)} = \lim_{x\to\infty} \frac{\cancel{x}\sqrt{10-\cancel{\frac{5}{x^2}}}}{\cancel{x}\left(\cancel{\frac{2}{x}}-3\right)} = \boxed{\frac{\sqrt{10}}{-3}}$$

$x>0$, $|x| = x$

$\frac{5}{x^2} \to 0$, $\frac{2}{x} \to 0$

$$\lim_{x\to-\infty} \frac{\sqrt{10x^2-5}}{2-3x} = \lim_{x\to-\infty} \frac{\sqrt{x^2\left(10-\frac{5}{x^2}\right)}}{x\left(\frac{2}{x}-3\right)} = \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{10-\frac{5}{x^2}}}{x\left(\frac{2}{x}-3\right)}$$

$$= \lim_{x\to-\infty} \frac{|x|\sqrt{10-\frac{5}{x^2}}}{x\left(\frac{2}{x}-3\right)} = \lim_{x\to-\infty} \frac{-\cancel{x}\sqrt{10-\cancel{\frac{5}{x^2}}}}{\cancel{x}\left(\cancel{\frac{2}{x}}-3\right)}$$

$x<0$
$|x| = -x$

$\frac{5}{x^2} \to 0$, $\frac{2}{x} \to 0$

$$= -\frac{\sqrt{10}}{-3} = \boxed{\frac{\sqrt{10}}{3}}$$