1. Evaluate $\log_3 108 - \log_3 4$ $= \log_3\left(\frac{108}{4}\right)$

$\log_a a^x = x$

$= \log_3 (27)$

$= \log_3 3^3 = \boxed{3}$

2. Solve for x: $\log(x+3) + \log(x) = 1$

$\log_a b = x \rightarrow a^x = b$

$\log(x+3)(x) = 1$

$10^1 = (x+3)(x)$

$10 = x^2 + 3x$

$0 = x^2 + 3x - 10$

$0 = (x+5)(x-2) \longrightarrow$ $\cancel{x = -5}$ extraneous

$x = 2$

3. Solve for x: $\ln x - \ln(x+1) = \ln 2 + \ln 3$

$\ln \frac{x}{x+1} = \ln 6$

$\log_a x = \log_a y$

then $x = y$

$\frac{x}{x+1} = 6$

$x = 6(x+1)$

$x = 6x + 6$

$-5x = 6$

$\cancel{x = -\frac{6}{5}}$

$\boxed{\text{extraneous no solution}}$

4. Find $\lim_{x\to 2^+} \ln(\underbrace{x-2}_{0^+})$

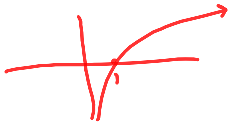
$= \ln(\text{"}0\text{"})$

$= -\infty$

$\lim_{x\to 0^+} \log_a x = -\infty$

$\lim_{x\to \infty} \log_a x = \infty$

$\lim_{x\to 1} \log_a x = 0$



5. Find $\lim_{x\to\infty} [\log(\underbrace{2x^2-1}_{\infty}) - \log(\underbrace{3x^2+6}_{\infty})] = \infty - \infty$ who knows!!

$$\lim_{x\to\infty} \log\left(\frac{2x^2-1}{3x^2+6}\right) = \log\left(\lim_{x\to\infty} \frac{2x^2-1}{3x^2+6} \to \frac{2}{3}\right)$$

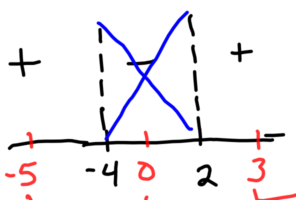
same degree
leading coefficients

$$= \boxed{\log \frac{2}{3}}$$

6. What is the domain of $f(x) = \ln(x^2 + 2x - 8)$?

solve $x^2 + 2x - 8 > 0$

$\underbrace{(x+4)(x-2)} > 0$



domain: $(-\infty, -4) \cup (2, \infty)$

7. Find $f'(x)$ for $f(x) = \ln(2x^2 - 8)$

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

chain Rule:

$\frac{d}{dx} \ln(2x^2-8) = \frac{1}{2x^2-8} \frac{d}{dx}(2x^2-8)$

$= \frac{1}{2x^2-8} (4x)$

$= \boxed{\frac{4x}{2x^2-8}}$

8. Find the derivative of $f(x) = 2^{\cos x} + \log_7(3x-1)$

$* \frac{d}{dx} a^x = a^x \ln a \; *$

$* \frac{d}{dx} \log_a x = \frac{1}{x \ln a} \; *$

$f'(x) = (2^{\cos x})(\ln a)(-\sin x) + \frac{1}{(3x-1)(\ln 7)} \cdot 3$

9. Find y' for $y = (\cos x)^{\tan x}$

$\log_a b^y = y \log_a b$

$\ln y = \ln(\cos x)^{\tan x}$

$\ln y = \tan x \ln(\cos x)$

$\frac{1}{y}\frac{dy}{dx} = \sec^2 x \ln(\cos x) + \tan x \left(\frac{-\sin x}{\cos x}\right)$

$\frac{dy}{dx} = y\left[\sec^2 x \ln(\cos x) + \tan x(-\tan x)\right]$

$= \boxed{(\cos x)^{\tan x}\left[\sec^2 x \ln(\cos x) - \tan^2 x\right]}$

10. Find the slope of the tangent line to the curve $f(x) = x\ln(x)$ at $x = e^2$.

$f'(x) = (1)\ln x + x\left(\frac{1}{x}\right)$

$f'(x) = \ln x + 1$

$m = f'(e^2)$

$= \ln e^2 + 1$

$= 2 + 1 = \boxed{3}$

$e^{\ln a} = a$

11. At a certain instant, 100 grams of a radioactive substance is present. After 4 years, 20 grams remain.

$y(t) = y_0 e^{kt}$ $y_0 =$ initial amount $y(4) = 20$

$y_0 = 100$

$y(t) = 100 e^{kt}$

$20 = 100 e^{k(4)}$

$\frac{1}{5} = e^{4k}$ → $\ln \frac{1}{5} = 4k$ → $\boxed{k = \frac{1}{4} \ln \frac{1}{5}}$

$y(t) = 100 e^{\left(\frac{1}{4}\ln\frac{1}{5}\right)t}$

$= 100 e^{\frac{t}{4}\ln\frac{1}{5}}$

$= 100 e^{\ln\left(\frac{1}{5}\right)^{\frac{t}{4}}}$

$\boxed{y(t) = 100\left(\frac{1}{5}\right)^{\frac{t}{4}}}$

b.) How much of the substance remains after 2.5 years?

$\boxed{y(2.5) = 100\left(\frac{1}{5}\right)^{\frac{2.5}{4}} \text{ grams}}$

a.) What is the half life of the substance?

at what time is $y(t) = \frac{1}{2} y_0 = \frac{1}{2}(100)$?

$y(t) = 100\left(\frac{1}{5}\right)^{\frac{t}{4}}$

$\frac{1}{2}(100) = 100\left(\frac{1}{5}\right)^{\frac{t}{4}}$ ⟶ $\frac{1}{2} = \left(\frac{1}{5}\right)^{\frac{t}{4}}$

$\ln \frac{1}{2} = \ln\left(\frac{1}{5}\right)^{\frac{t}{4}}$

$\ln \frac{1}{2} = \frac{t}{4} \ln \frac{1}{5}$

$t = \dfrac{4 \ln \frac{1}{2}}{\ln \frac{1}{5}}$ years

12. A bowl of soup at temperature 180° is placed in a 70° room. If the temperature of the soup is 150° after 2 minutes, when will the soup be an eatable 100°?

$y_0 =$ initial temp $= 180°$

$T =$ room temp $= 70°$

$y(2) = 150°$

solve $y(t) = 100°$

$$y(t) = (y_0 - T)e^{kt} + T$$

$$y(t) = (180-70)e^{kt} + 70$$

$$y(t) = 110e^{kt} + 70$$

$$150 = 110e^{k(2)} + 70$$

$$80 = 110e^{2k}$$

$$\frac{8}{11} = e^{2k}$$

$$\ln\frac{8}{11} = 2k$$

$$k = \frac{1}{2}\ln\frac{8}{11}$$

$$y(t) = 110e^{\left(\frac{1}{2}\ln\frac{8}{11}\right)t} + 70$$

$$y(t) = 110e^{\frac{t}{2}\ln\frac{8}{11}} + 70$$

$$\boxed{y(t) = 110\left(\frac{8}{11}\right)^{\frac{t}{2}} + 70}$$

solve $y(t) = 100$ for t

$$100 = 110\left(\frac{8}{11}\right)^{t/2} + 70$$

$$30 = 110\left(\frac{8}{11}\right)^{t/2}$$

$$\frac{3}{11} = \left(\frac{8}{11}\right)^{t/2}$$

$$\ln\frac{3}{11} = \frac{t}{2}\ln\left(\frac{8}{11}\right)$$

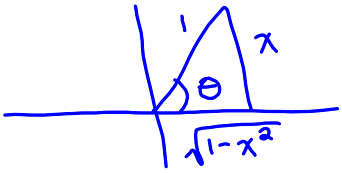
$$t = \frac{2\ln\frac{3}{11}}{\ln\frac{8}{11}} \text{ minutes}$$

13. Express $\tan(\underbrace{\arcsin x}_{\theta})$ as an algebraic expression.

let $\theta = \arcsin x$

$\sin\theta = \frac{x}{1} = \frac{opp}{hyp}$

$\sin\theta = x$



Find $\tan\theta$

$\boxed{\tan\theta = \frac{opp}{adj} = \frac{x}{\sqrt{1-x^2}}}$

$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

14. Find the derivative of $y = x^2 \arccos(e^{3x})$

product rule & chain rule

$\frac{dy}{dx} = 2x\arccos(e^{3x}) + x^2\left[-\frac{1}{\sqrt{1-(e^{3x})^2}} \cdot 3e^{3x}\right]$

15. Find the equation of the line tangent to

$y = \tan^{-1}(2x-1)$ when $x = 1$.

point: when $x = 1$, $y = \tan^{-1}(1) = \frac{\pi}{4}$ point: $(1, \frac{\pi}{4})$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

$m = \frac{d}{dx}\Big|_{x=1}$

$y = \tan^{-1}(2x-1)$

$\frac{dy}{dx} = \frac{1}{1+(2x-1)^2}(2)$

$\boxed{y - \frac{\pi}{4} = 1(x-1)}$

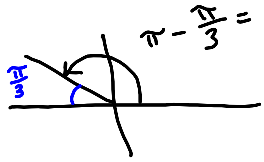
$m = \frac{1}{2}(2) = 1$

16. Compute the exact value of $\lim_{x\to\infty} \arccos\left(\frac{1+2x}{5-4x}\right)$ $= \arccos\left(\lim_{x\to\infty} \frac{1+2x}{5-4x}\right) = \frac{2\pi}{3}$

$\theta = \arccos\left(-\frac{1}{2}\right)$

$\cos\theta = -\frac{1}{2}, \quad 0 \le \theta \le \pi$

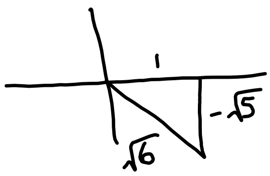
$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$



17. Compute $\sec(\underbrace{\arctan(-\sqrt{5})}_{\theta})$

$\theta = \arctan(-\sqrt{5})$

$\tan\theta = \frac{-\sqrt{5}}{1} \; \frac{opp}{adj} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

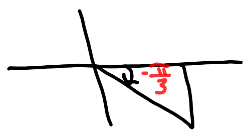


so, $\sec\theta = \frac{hyp}{adj} = \frac{\sqrt{6}}{1} = \boxed{\sqrt{6}}$

18. Compute $\arcsin(\sin\frac{4\pi}{3})$ $= \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$

$\theta = \arcsin\left(-\frac{\sqrt{3}}{2}\right)$

$\sin\theta = -\frac{\sqrt{3}}{2}, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



19. Find the limits of each of the following:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0} \ \underline{\text{or}} \ \frac{\infty}{\infty}$$

$$\text{then } \lim_{x\to a} \frac{f(x)}{g(x)} \overset{L}{=} \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

a) $\lim_{x\to 0} \frac{\arcsin(3x)}{2x}$ $\frac{0}{0}$ ✓

$$\overset{L}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-9x^2}}(3)}{2}$$

$$= \boxed{\frac{3}{2}}$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

b) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} = \frac{\infty}{\infty}$

$$\frac{\sqrt{x}}{x} = \frac{x^{\frac{1}{2}}}{x} = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$$

$$\overset{L}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \left(\frac{1}{x}\right)\left(\frac{2\sqrt{x}}{1}\right) = \lim_{x\to\infty} \frac{2}{\sqrt{x}}$$

$$= 0$$

c) $\lim_{x\to 0^+} \frac{\ln x}{\sqrt{x}} = \frac{-\infty}{0}$

$$= (-\infty)\left(\frac{1}{0}\right)$$

$$= (-\infty)(\infty) = \boxed{-\infty}$$

d.) $\lim_{x\to\pi/2^-}(\sec x - \tan x) = \lim_{x\to\frac{\pi}{2}^-}\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) = \infty - \infty = ?$

$$= \lim_{x\to\frac{\pi}{2}^-} \frac{1-\sin x}{\cos x} \quad \frac{0}{0}$$

$$\stackrel{L}{=} \lim_{x\to\frac{\pi}{2}^-} \frac{-\cos x}{-\sin x} = \frac{0}{-1} = \boxed{0}$$

indeterminate product is $(0)(\infty)$

"bring one down"

e.) $\lim_{x\to1^+}(x-1)\tan(\pi x/2) = (0)(\infty) \to$ bring one down!

① $\cancel{\lim_{x\to1^+} \frac{\tan\left(\frac{\pi x}{2}\right)}{\frac{1}{x-1}}}$ OR ② $\lim_{x\to1^+} \frac{x-1}{\frac{1}{\tan\left(\frac{x\pi}{2}\right)}}$

$$\lim_{x\to1^+} \frac{x-1}{\cot\left(\frac{\pi x}{2}\right)} = \frac{0}{0}$$

$\csc\frac{\pi}{2} = \frac{1}{\sin\frac{\pi}{2}} = 1$

$$\stackrel{L}{=} \lim_{x\to1^+} \frac{1}{-\csc^2\left(\frac{\pi x}{2}\right)\left(\frac{\pi}{2}\right)}$$

$$= \frac{-1}{\frac{\pi}{2}} = \boxed{\frac{-2}{\pi}}$$

f.) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{4x} = 1^\infty$

step1: $y = \left(1+\frac{2}{x}\right)^{4x}$

step2: $\ln y = \ln\left(1+\frac{2}{x}\right)^{4x}$

$\ln y = 4x\ln\left(1+\frac{2}{x}\right)$

Step3 $\lim_{x\to\infty}\ln y = \lim_{x\to\infty} 4x\ln\left(1+\frac{2}{x}\right) = \infty\ln(1)$
$= (\infty)(0)$ indeterminate product

Bring one down!

$$\lim_{x\to\infty} 4\underline{x}\ln\left(1+\frac{2}{x}\right)$$

$$\lim_{x\to\infty} \frac{4\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}} \quad \frac{0}{0}$$

$$\lim_{x\to\infty} \frac{4\left(\frac{1}{1+\frac{2}{x}}\right)\left(\frac{-2}{x^2}\right)}{\frac{-1}{x^2}}$$

$$\to \lim_{x\to\infty} 4\left(\frac{1}{1+\frac{2}{x}}\right)\left(\frac{+2}{\cancel{x^2}}\right)\left(\frac{+\cancel{x^2}}{1}\right)$$

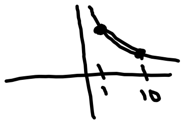
$$\lim_{x\to\infty} \frac{8}{1+\frac{2}{x}} = 8$$

step 3: e^{answer}

$\ln y = 8$

$\boxed{y = e^8}$

20. If $f(x) = \frac{1}{x}$, verify $f(x)$ satisfies the Mean Value Theorem on the interval $[1, 10]$ and find all c that satisfies the conclusion of the Mean Value Theorem.



MVT: there exists a c, $1 \le c \le 10$ so that

$$f'(c) = \frac{f(10) - f(1)}{10 - 1} = \frac{\frac{1}{10} - 1}{9} \cdot \frac{10}{10} = \frac{-9}{90} = \frac{-1}{10}$$

solve $f'(x) = -\frac{1}{10} \to \frac{-1}{x^2} = \frac{-1}{10} \to 10 = x^2$

$x = \sqrt{10}$

21. Find the absolute maximum and minimum of the given function on the given interval.

a) $x^3 - 5x^2 + 3$ on $[-1, 3]$

step 1: Find all critical numbers in $[-1, 3]$

$f'(x) = 3x^2 - 10x$

$= x(3x - 10)$

cn: $x = 0$

$\cancel{x = \frac{10}{3}}$

step 2: evaluate $f(x)$ at endpoints & c.n.

$f(-1) = -1 - 5 + 3 = -3$

$f(3) = 27 - 45 + 3 = -15$

$f(0) = 3$

$\boxed{\text{abs max} = 3 \quad \text{abs min} = -15}$

b) $x \ln x$ on $[1, e]$

$f'(x) = \ln x + x \cdot \frac{1}{x}$

$f'(x) = \ln x + 1$

cn: $\ln x + 1 = 0$

$\ln x = -1 \to x = e^{-1} = \cancel{\frac{1}{e}}$

$f(1) = \ln(1) = 0$

$f(e) = e \ln e = e$

$\boxed{\text{abs min} = 0 \quad \text{abs max} = e}$

22. Find the intervals where the given function is increasing and decreasing, local extrema, intervals of concavity and inflection points.

a) $f(x) = x^3 - 2x^2 + x$

$f'(x) = 3x^2 - 4x + 1$

$= (3x-1)(x-1)$

cn: $x = \frac{1}{3}$, $x = 1$

$+ \;|\; - \;|\; +$

$0 \quad \frac{1}{3} \quad \frac{3}{4} \quad 1 \quad 2$

inc: $(-\infty, \frac{1}{3}) \cup (1, \infty)$

dec: $(\frac{1}{3}, 1)$

max: $(\frac{1}{3}, \frac{4}{27})$

min: $(1, 0)$

$\frac{1}{27} - \frac{2}{9} + \frac{1}{3}$

$\frac{1-6+9}{27}$

$f''(x) = 6x - 4$

$f''(x) = 0$

$6x - 4 = 0$

$x = \frac{2}{3}$

$- \;|\; +$

$0 \quad \frac{2}{3} \quad 1$

concave up: $(\frac{2}{3}, \infty)$

concave down: $(-\infty, \frac{2}{3})$

inf pt: $(\frac{2}{3}, \frac{2}{27})$

b) $f(x) = xe^{2x}$

inc/dec/max/min: Look at $f'(x) = 1 \cdot e^{2x} + x \cdot 2e^{2x}$

$= e^{2x}(1+2x)$

cn: $x = -\frac{1}{2}$

e^{2x}: $+ \;|\; +$

$1+2x$: $- \;|\; +$

$\ominus \;|\; \oplus$

$-1 \quad -\frac{1}{2} \quad 0$

dec: $(-\infty, -\frac{1}{2})$

inc $(-\frac{1}{2}, \infty)$

local max none

local min: $(-\frac{1}{2}, -\frac{1}{2}e^{-1})$

$f(-\frac{1}{2})$

concavity & inflection points look at $f''(x)$

$f'(x) = e^{2x} + 2xe^{2x}$

$f''(x) = 2e^{2x} + 2e^{2x} + 2xe^{2x}(2)$

$= 4e^{2x} + 4xe^{2x}$

$f''(x) = 4e^{2x}(1+x)$

$f''(x) = 0$ if $x = -1$

$4e^{2x}$: $+ \;|\; +$

$1+x$: $- \;|\; +$

ccd $\ominus \;|\; \oplus$ + ccu

$-2 \quad -1 \quad 0$

concave up: $(-1, \infty)$

concave down: $(-\infty, -1)$

inflection point: $(-1, -e^{-2})$

23. Find the concavity of f if $f'(x) = \dfrac{\ln x}{x}$

$$f''(x) = \frac{\frac{1}{x}(x) - \ln x\,(1)}{x^2}$$

$$f''(x) = \frac{1 - \ln x}{x^2}$$

$f''(x) = 0$ if $1 - \ln x = 0$

$1 = \ln x$

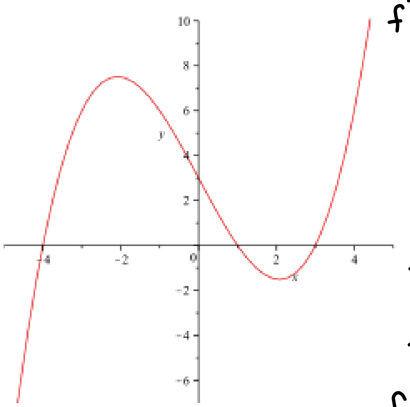
$x = e$

$+ \;|\; -$

$0 \quad 1 \quad e \quad e^2$

$\boxed{\begin{array}{l} \text{cc up: } (0, e) \\ \text{ccdown: } (e, \infty) \end{array}}$

24. In the graph that follows, the graph of f' is given. Using the graph of f', determine all critical values of f, where f is increasing and decreasing, local extrema of f, where f is concave up and concave down, and the x-coordinates of the inflection points of f. Assume f is continuous.



f'

cn: $f' = 0$ $\quad x = -4, 1, 3$

f inc: $f' > 0 \to (-4, 1) \cup (3, \infty)$

f dec: $f' < 0 \to (-\infty, -4) \cup (1, 3)$

f local max: $x = 1$

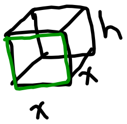
f local min: $x = -4$, $x = 3$

f concave up: f' inc $(-\infty, -2) \cup (2, \infty)$

f concave down: f' dec $(-2, 2)$

inflection points: $x = -2$, $x = 2$

25. A cardboard rectangular box holding 32 cubic inches with a square base and open top is to be constructed. If the material for the base costs \$2 per square inch and material for the sides costs \$5 per square inch, find the dimensions of the cheapest such box.



constraint: $V = 32$

$\boxed{x^2 h = 32} \rightarrow h = \dfrac{32}{x^2}$

each side is xh in^2

minimize $\text{cost} = C_{\text{base}} + C_{4\text{ sides}}$

$= 2(x^2) + 5(4xh)$

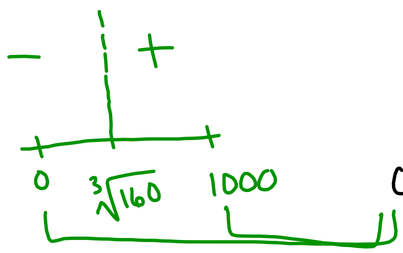
$= 2x^2 + 20xh$

$= 2x^2 + 20x\left(\dfrac{32}{x^2}\right)$

$C = 2x^2 + \dfrac{640}{x}$ Find minimum

$C' = 4x - \dfrac{640}{x^2}$

$C' = \dfrac{4x^3 - 640}{x^2}$



cn:

$4x^3 - 640 = 0$

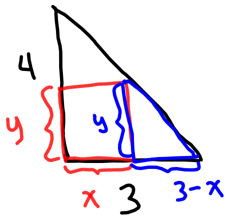
$4x^3 = 640$

$x^3 = 160$

$x = \sqrt[3]{160}$

$$\boxed{\begin{aligned} h &= \frac{32}{x^2} \text{ in} \\ h &= \frac{32}{(\sqrt[3]{160})^2} \text{ in} \end{aligned}}$$

26. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of length 3 m and 4 m if two sides of the rectangle lie along the legs.



maximize $A_{\text{rectangle}} = xy$

$$\frac{y}{3-x} = \frac{4}{3}$$

$$y = \frac{4}{3}(3-x)$$

$$A = x \cdot \frac{4}{3}(3-x)$$

$$= \frac{4}{3}x(3-x)$$

$$A = 4x - \frac{4}{3}x^2$$

$$A' = 4 - \frac{8}{3}x$$

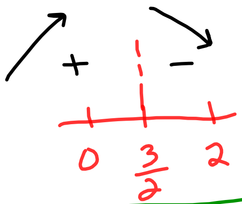
$$A' = 0 \longrightarrow \quad 4 - \frac{8}{3}x = 0$$

$$4 = \frac{8}{3}x$$

$$\boxed{x = \frac{12}{8} = \frac{3}{2}}$$

get area

$$y = \frac{4}{3}\left(3 - \frac{3}{2}\right)$$



$$\boxed{\text{Area} = \frac{4}{3}\left(\frac{3}{2}\right)\left(3 - \frac{3}{2}\right)} \ m^2$$

27. Find an antiderivative of $\dfrac{1}{\sqrt{1-x^2}} - \dfrac{1+x}{x}$. $= \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{x} - 1$

$F(x) = \arcsin x - \ln|x| - x + C$

28. Given $f''(x) = 2e^x - 4\sin(x)$, $f(0) = 1$, and $f'(0) = 2$, find $f(x)$.

$f''(x) = 2e^x - 4\sin(x)$

$f'(x) = 2e^x + 4\cos x + C$

$f'(0) = 2$ $\quad 2 = 2e^0 + 4\cos(0) + C$

$2 = 6 + C \rightarrow \boxed{C = -4}$

$f'(x) = 2e^x + 4\cos x - 4$

$f(x) = 2e^x + 4\sin x - 4x + \cancel{d}^{\,-1}$

$f(0) = 1$

$1 = 2e^0 + 4\sin(0) - 4(0) + d$

$1 = 2 + d$ $\quad \boxed{d = -1}$

29. Find the vector functions that describe the velocity and position of a particle that has an acceleration of $\mathbf{a}(t) = \langle \sin t, 2\rangle$, initial velocity of $\mathbf{v}(0) = \langle 1, -1\rangle$ and an initial position of $\mathbf{r}(0) = \langle 0, 0\rangle$.

$a(t) = \langle \sin t, 2\rangle$

$v(t) = \langle -\cos t + c_1, 2t + c_2\rangle$

$v(0) = \langle 1, -1\rangle \longrightarrow \langle 1, -1\rangle = \langle -\cos(0) + c_1, 0 + c_2\rangle$

$\langle 1, -1\rangle = \langle -1 + c_1, c_2\rangle \longrightarrow$ $1 = -1 + c_1$ $\boxed{c_1 = 2}$ $\boxed{c_2 = -1}$

$v(t) = \langle -\cos t + 2, 2t - 1\rangle$

$r(t) = \langle -\sin t + 2t + c_3, t^2 - t + c_4\rangle$

$r(0) = \langle 0, 0\rangle \rightarrow \langle 0, 0\rangle = \langle c_3, c_4\rangle$ $\quad \boxed{c_3 = 0 \quad c_4 = 0}$

30. $\sum_{i=2}^{5} i^2 =$ $2^2 + 3^2 + 4^2 + 5^2$

31. Write $1 + \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \frac{1}{e^5}$ in summation notation.

$$\sum_{i=0}^{5} \frac{1}{e^i}$$

32. $\sum_{i=3}^{99} \left(\frac{1}{i} - \frac{1}{i+1}\right) =$ $\underbrace{\left(\frac{1}{3} - \cancel{\frac{1}{4}}\right)}_{i=3} + \underbrace{\left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right)}_{i=4} + \underbrace{\left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{6}}\right)}_{i=5} + \dots + \underbrace{\left(\cancel{\frac{1}{99}} - \frac{1}{100}\right)}_{i=100}$

$$= \frac{1}{3} - \frac{1}{100} = \frac{97}{300}$$