Definition: If $y(t)$ is the value of a quantity at time t and if the rate of change of y with respect to t is proportional to its size $y(t)$ at any time, $\underbrace{\text{[That is } \frac{dy}{dt} = ky]}$, then the quantity $y(t)$ at time t is given by

$$\boxed{y(t) = y_0 e^{kt}}$$ $y_0 =$ initial amount

Section 4.5

1. A bacteria culture starts with 400 bacteria and the population triples every 20 minutes.

   a.) Find an expression for the number of bacteria after t hours.

20 min $= \frac{1}{3}$ hour

$y(t) = y_0 e^{kt}$ $y_0 = 400$

given $y(\frac{1}{3}) = 3(400)$

$3(\cancel{400}) = \cancel{400} e^{k(\frac{1}{3})}$

$3 = e^{\frac{1}{3}k}$

$\ln 3 = \frac{1}{3} k$

$a \ln x = \ln x^a$

$3 \ln 3 = k$

$\ln 27 = k$

$y(t) = 400 e^{(\ln 27)t}$

Recall: $e^{\ln a} = a$

$y(t) = 400 e^{t \ln 27}$

$y(t) = 400 e^{\ln(27)^t}$

$$\boxed{y(t) = 400(27)^t \quad t \text{ in hours}}$$

b.) Find the number of bacteria after 2 days.

2 days = 48 hours

$$\boxed{y(48) = 400(27)^{48} \text{ bacteria}}$$

c.) When will the population reach 20,000?

$y(t) = 400(27)^t$ solve $y(t) = 20{,}000$ for t.

$\frac{20{,}000}{400} = \frac{400(27)^t}{400}$

$50 = (27)^t$

$\ln 50 = \ln(27)^t$

$\ln 50 = t \ln(27)$

$$\boxed{t = \frac{\ln 50}{\ln 27} \text{ hours}}$$

2. Polonium-210 has a half-life of 140 days. If a sample has a mass of 200 mg, find a formula for the mass that remains after t days.

half life = how long it takes for half the substance to disintegrate.

$y(t) = y_0 e^{kt}$

$y(t) = 200 e^{kt}$ $\qquad y(140) = \frac{1}{2}(200) = 100$

$100 = 200 e^{k(140)}$

$\frac{1}{2} = e^{140k} \rightarrow \ln\frac{1}{2} = 140k \rightarrow \boxed{k = \frac{1}{140}\ln\frac{1}{2}}$

$y(t) = 200 e^{kt}$

$y(t) = 200 e^{\left(\frac{1}{140}\ln\frac{1}{2}\right)t}$

$y(t) = 200 e^{\frac{t}{140}\ln\frac{1}{2}}$

$y(t) = 200 \cancel{e}^{\cancel{\ln}\left(\frac{1}{2}\right)^{\frac{t}{140}}}$

$\boxed{y(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{140}}}$

3. After 3 days a sample of radon-222 decayed to 58% of its original amount. What is the half-life of radon-222?

original amount $\rightarrow y_0$

$y(t) = y_0 e^{kt}$

given $y(3) = .58 y_0$

$.58 \cancel{y_0} = \cancel{y_0} e^{k(3)}$

$.58 = e^{3k} \rightarrow \ln(.58) = 3k \rightarrow k = \frac{1}{3}\ln(.58)$

$y(t) = y_0 e^{\frac{t}{3}\ln(.58)}$

$y(t) = y_0 \cancel{e}^{\cancel{\ln}(.58)^{\frac{t}{3}}}$

$\boxed{y(t) = y_0(.58)^{\frac{t}{3}}}$

Now, find the half-life

solve $y(t) = \frac{1}{2}y_0$

$\frac{1}{2}\cancel{y_0} = \cancel{y_0}(.58)^{\frac{t}{3}}$

$\frac{1}{2} = (.58)^{\frac{t}{3}}$

$\ln\frac{1}{2} = \ln(.58)^{\frac{t}{3}}$

$\ln\frac{1}{2} = \frac{t}{3}\ln(.58)$

$\boxed{\frac{3\ln\frac{1}{2}}{\ln(.58)} = t \text{ days}}$

4. The rate of change of atmospheric pressure P with respect to altitude h is proportional to P, provided that the temperature is constant. At a specific temperature the pressure is 101 kPa at sea level and 86.9 kPa at h = 1,000 m. What is the pressure at an altitude of 3500 m?

$$\frac{dy}{dt} = ky$$

then $y(t) = y_0 e^{kt}$

$$\frac{dP}{dh} = kP$$

$P_0 =$ pressure when $h=0$

$P_0 = 101$

$P(1000) = 86.9$

$$P(h) = P_0 e^{kh}$$

$$P(h) = 101\, e^{kh}$$

$$86.9 = 101\, e^{k(1000)}$$

$$\frac{86.9}{101} = e^{1000k}$$

$$\ln \frac{86.9}{101} = 1000k$$

$$k = \frac{1}{1000} \ln \frac{86.9}{101}$$

$$P(h) = 101\, e^{\frac{h}{1000} \ln \frac{86.9}{101}}$$

$$P(h) = 101 \left(\frac{86.9}{101}\right)^{\frac{h}{1000}}$$

so $P(3500) = 101 \left(\frac{86.9}{101}\right)^{\frac{3500}{1000}}$ KPa

5. A curve that passes through the point $(0, 25)$ has the property that the slope at every point (x, y) is eight times the y coordinate. Find the equation of the curve.

$$\frac{dy}{dx} = 8y \longrightarrow y(x) = y_0 e^{8x}$$

$y(0) = 25$

$y_0 = 25$

$$\boxed{y(x) = 25\, e^{8x}}$$

Definition: The rate of cooling of an object is proportional to the temperature difference between the object and the temperature of the object's surroundings. If $y(t)$ is the temperature of the object at time t, then $\frac{dy}{dt} = k(y - T)$, where y is the temperature of the object at time t and T is the room temperature (the temperature of the room in which the object is cooling). The solution of this equation, which gives the temperature of the object at time t, is $y(t) = (y_0 - T)e^{kt} + T$, where y_0 is the initial temperature of the object.

$y_0 =$ initial temp of object

$T =$ cooling room temp

6. A pie is taken from an oven, where the temperature is 450°, to a 75° room. After 15 minutes, the temperature of the pie reads 350°. What will the temperature of the pie be after 27 minutes?

$y_0 = 450°$

$T = 75°$

$y(15) = 350°$

$$y(t) = (450 - 75)e^{kt} + 75$$

$$y(t) = 375e^{kt} + 75$$

$$350 = 375e^{k(15)} + 75$$

$$350 - 75 = 375e^{15k}$$

$$\frac{275}{375} = \frac{375e^{15k}}{375}$$

$$\frac{11}{15} = e^{15k} \rightarrow \ln\frac{11}{15} = 15k$$

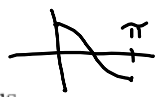
$$\frac{1}{15}\ln\frac{11}{15} = k$$

$$y(t) = 375e^{kt} + 75$$

$$y(t) = 375e^{\frac{t}{15}\ln\left(\frac{11}{15}\right)} + 75$$

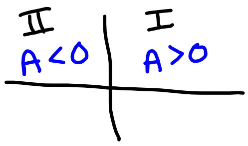
$$y(t) = 375\left(\frac{11}{15}\right)^{\frac{t}{15}} + 75$$

$$\boxed{y(27) = 375\left(\frac{11}{15}\right)^{\frac{27}{15}} + 75 \text{ degrees}}$$



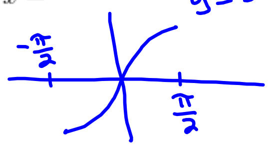
I. INVERSE COSINE: If $\boxed{0 \leq x \leq \pi,}$ then $f(x) = \cos x$ is one-to-one, thus the inverse exists, denoted by $\cos^{-1}(x)$ or $\arccos x$. Additionally, the domain of $\arccos x$ = range of $\cos x = [-1, 1]$ and range of $\arccos x$ = domain of $\cos x = [0, \pi]$. Note: $\arccos(x)$ is the **angle** in $[0, \pi]$ whose cosine is x.

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \longrightarrow \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

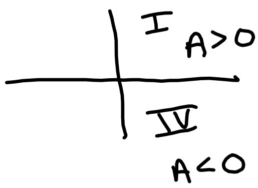


To solve $\arccos(A)$, Find θ so that $\cos(\theta) = A$
$0 \leq \theta \leq \pi$

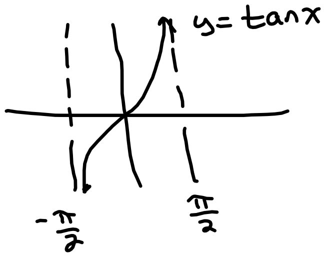
II. INVERSE SINE: If $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, then $f(x) = \sin x$ is one-to-one, thus the inverse exists, denoted by $\sin^{-1}(x)$ or $\arcsin x$. Additionally, the domain of $\arcsin x$ = range of $\sin x = [-1, 1]$ and range of $\arcsin x$ = domain of $\sin x = [-\frac{\pi}{2}, \frac{\pi}{2}]$. Note: $\arcsin(x)$ is the **angle** in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ whose sine is x.



to find $\arcsin(A)$, Find θ so that
$\sin(\theta) = A$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$



III. INVERSE TANGENT: If $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then $f(x) = \tan x$ is one-to-one, thus the inverse exists, denoted by $\tan^{-1}(x)$ or $\arctan x$. Additionally, the domain of $\arctan x$ = range of $\tan x = (-\infty, \infty)$ and range of $\arctan x$ = domain of $\tan x = (-\frac{\pi}{2}, \frac{\pi}{2})$. Note: $\arctan(x)$ is the **angle** in $(-\frac{\pi}{2}, \frac{\pi}{2})$ whose tangent is x.



to find $\arctan A$, find θ
so that $\tan\theta = A$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

reference angle →

θ	0° (0)	30° ($\frac{\pi}{6}$)	45° ($\frac{\pi}{4}$)	60° ($\frac{\pi}{3}$)	90° ($\frac{\pi}{2}$)
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\frac{\sin\theta}{\cos\theta} = \tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

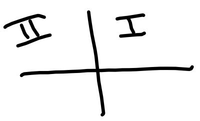
Section 4.6

7. Compute the following without the aid of a calculator.

a.) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ since $\frac{\sqrt{3}}{2} > 0$, go with reference angle

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

b.) $\arccos\left(-\frac{1}{\sqrt{2}}\right)$



note: $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

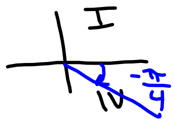
$\arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

since $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$,

put in Q II by solving

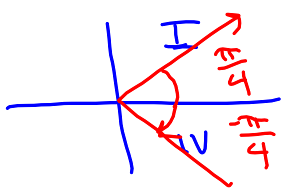
$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

c.) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$



$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

d.) $\arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$ What if $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$



$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \theta$

$\sin\theta = -\frac{\sqrt{2}}{2}$, $\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

* $\arccos\frac{1}{2} = \frac{\pi}{3}$

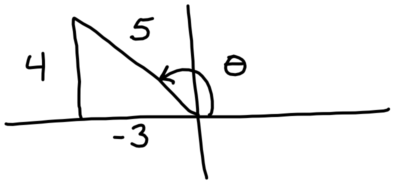
* $\text{arssin}\,\frac{1}{2} = \frac{\pi}{6}$

* $\arccos\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

e.) $\cot\left(\underbrace{\arccos\left(-\frac{3}{5}\right)}\right)$

let $\theta = \arccos\left(-\frac{3}{5}\right)$



$\cos\theta = -\frac{3}{5}\ \frac{\text{adj}}{\text{hyp}}$

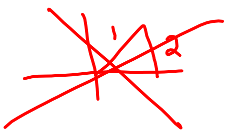
↑ negative

$\theta = \text{II}$ angle

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{-3}{4}$

f.) $\sin(\arcsin 2)$

let $\theta = \arcsin 2 \rightarrow \sin\theta = 2$ no solution since the range of $\sin x$ is $[-1, 1]$



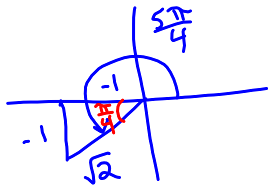
g.) $\arccos\left(\cos\left(\frac{2\pi}{3}\right)\right)$

$= \arccos\left(-\frac{1}{2}\right)$

$= \frac{2\pi}{3}$

h.) $\arctan\left(\tan\frac{5\pi}{4}\right)$

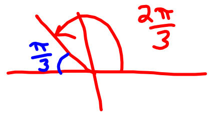
$\frac{5\pi}{4} = \pi + \frac{\pi}{4} \rightarrow$ QIII



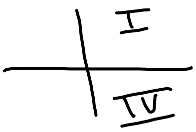
$\arctan(1) = \frac{\pi}{4}$

$\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$

$\frac{2\pi}{3} = \pi - \frac{\pi}{3}$



$\arctan(-\sqrt{3})$

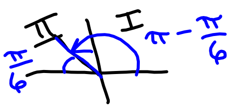


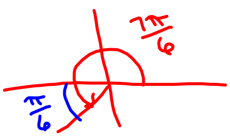
$\tan\frac{2\pi}{3} = -\sqrt{3}$

$\arccos\left(\cos\left(\frac{7\pi}{6}\right)\right)$

$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$ QIII

$\arccos\left(-\frac{\sqrt{3}}{2}\right)$





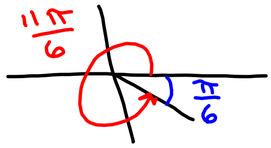
$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$= \boxed{\frac{5\pi}{6}}$

i.) $\arcsin\left(\sin\left(\frac{11\pi}{6}\right)\right)$

$\frac{11\pi}{6} = 2\pi - \frac{\pi}{6} \longrightarrow Q_{IV}$

$= \arcsin\left(-\frac{1}{2}\right)$



$\sin\frac{11\pi}{6} = -\frac{1}{2}$

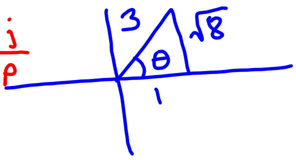
$= -\frac{\pi}{6}$

j.) $\sin\left(2\arccos\left(\frac{1}{3}\right)\right)$

θ

$\theta = \arccos\left(\frac{1}{3}\right)$

$\cos\theta = \frac{1}{3}\ \frac{\text{adj}}{\text{hyp}}$



$\sin(2\theta) = 2\sin\theta\cos\theta$

$= \boxed{2\left(\frac{\sqrt{8}}{3}\right)\left(\frac{1}{3}\right)}$

Derivatives of Inverse Trigonometric Functions:

A.) $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$.

B.) $\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$.

C.) $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

Derivatives of Inverse Trigonometric Functions:

A.) $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$.

B.) $\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$.

C.) $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

8. Find the derivative of $y = \arctan(1-x)$

$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$ by chain rule,

$$\frac{d}{dx}\arctan(1-x) = \frac{1}{1+(1-x)^2}\cdot\frac{d}{dx}(1-x)$$

$$= \boxed{\frac{1}{1+(1-x)^2}(-1)}$$

9. Find the equation of the tangent line to the graph of $y = \arcsin\frac{x}{2}$ at $x = -1$.

$$m = \frac{d}{dx}\arcsin\frac{x}{2}\Big|_{x=-1}$$

chain rule gives

$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\arcsin\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\frac{1}{2}$

let $x=-1$:

$$m = \frac{1}{\sqrt{1-\left(\frac{-1}{2}\right)^2}}\cdot\frac{1}{2} = \frac{1}{\sqrt{\frac{3}{4}}}\cdot\frac{1}{2}$$

$$= \frac{2}{\sqrt{3}}\cdot\frac{1}{2} = \frac{1}{\sqrt{3}}$$

Thus $m = \frac{1}{\sqrt{3}}$

point: $x=-1$, $y = \arcsin\left(-\frac{1}{2}\right)$

$y = -\frac{\pi}{6}$

$$\boxed{\text{equation: } y + \frac{\pi}{6} = \frac{1}{\sqrt{3}}(x+1)}$$

10. What is the domain of $f(x) = \arcsin(2x-1)$? Of $\arctan(2x-1)$?

domain of $\arcsin x$ = range of $\sin x$ $\quad -1 \le x \le 1$

$\therefore$ domain of $\arcsin(2x-1)$ is $\quad -1 \le 2x-1 \le 1$

$0 \le 2x \le 2$

$\boxed{0 \le x \le 1}$

domain of $\arctan x$ = range of $\tan x$

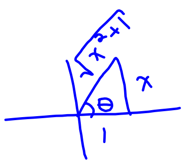
$= \mathbb{R}$ or $(-\infty, \infty)$

domain of $\arctan(2x-1)$ is $(-\infty, \infty)$

11. ~~cos(arctan x)~~ is equivalent to what?

let $\theta = \arctan x$

$\tan\theta = \frac{x}{1}\ \frac{\text{opp}}{\text{adj}}$



$$\boxed{\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{x^2+1}}}$$

Section 4.8: L'Hospital's Rule

Indeterminate form: If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then we say the limit is in indeterminate form.

L'Hospital's Rule: If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.

Section 4.8

12. Find the following limits.

a.) $\lim_{x\to\infty} \frac{(\ln x)^2}{x-1} = \frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x-1} \overset{L}{=} \lim_{x\to\infty} \frac{2(\ln x)\cdot\frac{1}{x}}{1}$$

$$= \lim_{x\to\infty} \frac{2\ln x}{x} = \frac{\infty}{\infty}$$

$$\overset{L}{=} \lim_{x\to\infty} \frac{2\cdot\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{2}{x}$$

$$= \boxed{0}$$

b.) $\lim_{x\to 0} \frac{\sin x - x}{x^3} = \frac{0}{0}$

$$\overset{L}{=} \lim_{x\to 0} \frac{\cos x - 1}{3x^2} = \frac{0}{0}$$

$$\overset{L}{=} \lim_{x\to 0} \frac{-\sin x}{6x} = \frac{0}{0}$$

$$\overset{L}{=} \lim_{x\to 0} \frac{-\cos x}{6} = \frac{-1}{6}$$

or, use $\lim_{x\to 0} \frac{\sin x}{x} = 1$

to get $\lim_{x\to 0} \frac{-\sin x}{6x} = \frac{-1}{6}$

Indeterminate Products: If $\lim_{x \to a} f(x)g(x) = 0 \cdot \infty$, this limit is an indeterminate product. Why do we call the product indeterminate?

$\lim_{x \to \infty} \frac{1}{x^2} \cdot x = (0)(\infty)$ $\qquad$ $\lim_{x \to \infty} \frac{1}{x} \cdot x^2 = 0 \cdot \infty$ $\qquad$ $\lim_{x \to \infty} \frac{1}{x^2} \cdot 6x^2 = 0 \cdot \infty$

$\lim_{x \to \infty} \frac{1}{x^2} x = \lim_{x \to \infty} \frac{1}{x} = 0$ $\qquad$ $\lim_{x \to \infty} \frac{1}{x} \cdot x^2 = \lim_{x \to \infty} (x) = \infty$ $\qquad$ $= \lim_{x \to \infty} \frac{1}{x^2} \cdot 6x^2 = 6$

All three of these limits are of the form $0 \cdot \infty$, yet they all have different limits. The goal is ty try to manipulate the product get the limit in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then use L'Hospital's rule.

c.) $\lim_{x \to 0^+} x^2 \ln x = (0)(-\infty)$

$$\lim_{x \to 0^+} x^2 \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}} \qquad \boxed{\frac{-\infty}{\infty}}$$

$$\frac{d}{dx} \frac{1}{x^2} = \frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

$$\stackrel{L}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}}$$

$$= \lim_{x \to 0^+} \frac{1}{x} \left(\frac{x^3}{-2} \right)$$

$$= \lim_{x \to 0^+} \frac{-x^2}{2} = \boxed{0}$$

Indeterminate Powers: If $\lim_{x\to a} f(x)^{g(x)}$ is of the form 0^0, ∞^0 or 1^∞, then the limit is an indeterminate power. To solve such a limit, take the natural logarithm, which converts the indeterminate power into an indeterminate product.

d.) $\lim_{x\to\infty}(e^x+x)^{\frac{1}{x}} = (\infty)^0$

let $y = (e^x+x)^{\frac{1}{x}}$

$\ln y = \ln(e^x+x)^{\frac{1}{x}}$

$\ln y = \frac{1}{x}\ln(e^x+x)$

$\ln y = \frac{\ln(e^x+x)}{x}$

$\lim_{x\to\infty} \frac{\ln(e^x+x)}{x} \quad \frac{\infty}{\infty}$

$\overset{L}{=} \lim_{x\to\infty} \frac{\frac{e^x+1}{e^x+x}}{1} \quad \frac{\infty}{\infty}$

$\overset{L}{=} \lim_{x\to\infty} \frac{e^x}{e^x+1} \quad \frac{\infty}{\infty}$

$\overset{L}{=} \lim_{x\to\infty} \frac{e^x}{e^x} = 1$

$\ln y \to 1$ so $\boxed{y \to e^1}$

e.) $\lim_{x\to 0}(\sin x)^{\tan x} = 0^0$

$y = (\sin x)^{\tan x}$

$\ln y = \ln(\sin x)^{\tan x}$

$\ln y = \tan x \ln(\sin x)$

$\lim_{x\to 0} \tan x \ln(\sin x) = (0)(\ln(0))$

$= (0)(-\infty) \longrightarrow$ indeterminate product

turn it into a fraction

$= \lim_{x\to 0} \frac{\ln(\sin x)}{\frac{1}{\tan x}} = \frac{-\infty}{\infty}$

$= \lim_{x\to 0} \frac{\ln(\sin x)}{\cot x}$

$\overset{L}{=} \lim_{x\to 0} \frac{\frac{\cos x}{\sin x}}{-\csc^2 x}$

$= \lim_{x\to 0} \frac{\frac{\cos x}{\sin x}}{\frac{-1}{\sin^2 x}}$

$= \lim_{x\to 0} \frac{\cos x}{\sin x}(-\sin^2 x)$

$= \lim_{x\to 0} (-\sin x \cos x)$

$= 0$

so $\ln y \to 0$

$y \to e^0 = 1$

f.) $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) = \frac{1}{0} - \frac{1}{0} = \infty - \infty \longrightarrow \frac{?}{0}$

$$= \lim_{x\to 1} \frac{x-1-\ln x}{(\ln x)(x-1)} = \frac{0}{0}$$

$$\overset{L}{=} \lim_{x\to 1} \frac{1-\frac{1}{x}}{\frac{1}{x}(x-1)+\ln x} = \lim_{x\to 1} \frac{1-\frac{1}{x}}{1-\frac{1}{x}+\ln x} \quad \frac{0}{0}$$

$$\overset{L}{=} \lim_{x\to 1} \frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{1}{x}} = \boxed{\frac{1}{2}}$$