Sections 1.1-1.3: Introduction to Differential Equations (PREP WORK)

(Answer the following questions using the electronic submission in Canvas-under “Prep Assignments”. You have unlimited submissions, so keep a record of your answers in case you decide to change any after our in-class discussion of the assignment. Prep Assignments will automatically be scored 10 out of 10, but will be spot-checked for legitimate attempted answers-failure to do so will result in a ZERO on the assignment!).

1. Read p1 and pp16-18 and define the following terms:
   (a) **Differential Equation**

   (b) **Ordinary Differential Equation**

   (c) **Partial Differential Equation**

   (d) **Order** of a Differential Equation

   (e) **Linear Differential Equation**

   (f) **Solution** to a Differential Equation

2. Read p3. Explain what a direction field is, and how to draw one.

3. Here is one differential equation you should have already seen: what is the solution to $\frac{dy}{dt} = a * y$?

Sections 1.1-1.3: Introduction to Differential Equations

Definitions:

- **Differential Equation-**

- **Ordinary Differential Equation**

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- **Solution** to a Differential Equation

Examples:

Verify that $y = Ce^{at}$ is a solution to the ODE $\frac{dy}{dt} = a * y$. If we also have the initial condition $y(0) = 10$, what else can we say about the solution? (NOTE: a differential equation with an initial condition is called an **initial value problem**)

Classify each of the differential equations (linear or nonlinear? order?)

- $y'' + 2y' - 3y = 0$

- $\frac{dy}{dt} = a * y^2$

- $t^2y''' - 4ty'' + 4y' = 0$

Find the values of r for which $y = t^r$ is a solution to the ODE $t^2 y''' - 4ty'' + 4y' = 0$.

Modelling with Differential Equations

Suppose an object is dropped from rest. What forces act on it? Use Newton's second law to create an initial value problem for the velocity of the object. Classify this equation as done in the previous example, and show that $y = \frac{mg}{c} - \frac{mg}{c} e^{-ct/m}$ is a solution to the IVP.

Direction Fields

Given a differential equation of the form $\frac{dy}{dt} = f(t, y)$, we can visualize the families of solutions using a **direction field**:

$y' = \frac{1}{e^t - y}$

$y' = -2y$