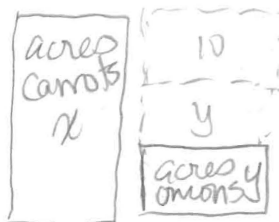


Part 2 - Systems of Linear Equations

① no soln \Rightarrow parallel lines \Rightarrow same slope & diff intercept

$3x - y = 4$ or $y = 3x - 4 \Rightarrow m = 3$
 $-6x + ky = 10$ or $y = (-6/k)x + 10/k \Rightarrow m = -6/k$
 $-6/k = 3 \Rightarrow k = -2$, check
 $10/k = 10/-2 = -5 \neq -4$ so diff y-int

② $x = \#$ of acres of carrots
 $y = \#$ of acres of onions
 $x + y = 100$ total acres
 $x = 2y + 10$ ratio carrots/onions
 Solve graphically, algebraically or with RREF
 $\Rightarrow x = 70, y = 30$
 Plant 30 acres of onions



③ $x = \#$ of shares in ABC
 $y = \#$ of shares in XYZ
 $32x + 23y = 10,100$ total money
 $1.2x + 1.4y = 540$ total div

$$\left[\begin{array}{cc|cc} 32 & 23 & 10100 & \\ 1.2 & 1.4 & 540 & \end{array} \right] \xrightarrow{\text{rref}}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 100 \\ 0 & 1 & 300 \end{array} \right] \begin{array}{l} x = 100 \\ y = 300 \end{array}$$

Invest 100 sh of ABC and 300 sh of XYZ

$x = \#$ of \$ in ABC
 $y = \#$ of \$ in XYZ
 $x + y = 10,100$ total money
 $\left(\frac{x}{32}\right)(1.2) + \left(\frac{y}{23}\right)(1.4) = 540$ total div

$$\left[\begin{array}{cc|cc} 1 & 1 & 10100 & \\ 1.2/32 & 1.4/23 & 540 & \end{array} \right] \xrightarrow{\text{rref}}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3200 \\ 0 & 1 & 6900 \end{array} \right] \begin{array}{l} x = 3200 \\ y = 6900 \end{array}$$

Invest \$3200 in ABC and \$6900 in XYZ

$$\textcircled{4} \begin{bmatrix} x & y & & \\ 0 & 1 & 0 & \\ 0 & 0 & & b \end{bmatrix} \text{ any value of } b$$

however if $b \neq 0$ then no solution
if $b = 0$, solution is $(t, 0)$ t any #

$$\textcircled{5} \begin{cases} 2x + 3y + z = 1 \\ x + y + z = 3 \\ 3x + 4y + 2z = 4 \end{cases} \leftrightarrow \begin{bmatrix} x & y & z & = & \# \\ 2 & 3 & 1 & | & 1 \\ 1 & 1 & 1 & | & 3 \\ 3 & 4 & 2 & | & 4 \end{bmatrix} \xrightarrow{\text{RREF}}$$

$$\begin{matrix} \textcircled{x} & \textcircled{y} & z \rightarrow t & \# \\ \textcircled{1} & 0 & 2 & 8 \\ 0 & \textcircled{1} & -1 & -5 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$\begin{bmatrix} \textcircled{1} & 0 & 2 & | & 8 \\ 0 & \textcircled{1} & -1 & | & -5 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \begin{cases} 1x + 0y + 2z = 8 \text{ or } x + 2t = 8 \\ 0x + 1y - 1z = -5 \text{ or } y - t = -5 \\ 0x + 0y + 0z = 0 \end{cases}$$

$$(x, y, z) = (8 - 2t, t - 5, t) \quad t \text{ any } \#$$

6. $-\frac{1}{2}x + 3y = 15$
 $2x + 5y = 8$ \rightarrow $\begin{bmatrix} -\frac{1}{2} & 3 & | & 15 \\ 2 & 5 & | & 8 \end{bmatrix}$ $\xrightarrow{-2R_1 \rightarrow R_1}$

$\begin{bmatrix} 1 & -6 & | & -30 \\ 2 & 5 & | & 8 \end{bmatrix}$ $\xrightarrow{-2R_1 + R_2 \rightarrow R_2}$ $\begin{bmatrix} 1 & -6 & | & -30 \\ 0 & 17 & | & 68 \end{bmatrix}$

$\frac{(1)(-2)}{0}$ $\frac{(-6)(-2)}{17}$ $\frac{(-30)(-2)}{68}$

$\frac{1}{17}R_2 \rightarrow R_2$ $\begin{bmatrix} 1 & -6 & | & -30 \\ 0 & 1 & | & 4 \end{bmatrix}$ $\xrightarrow{6R_2 + R_1 \rightarrow R_1}$

$\frac{(6)(6)}{1}$ $\frac{(1)(6)}{-6}$ $\frac{(4)(6)}{-30}$

$\begin{bmatrix} 1 & 0 & | & -6 \\ 0 & 1 & | & 4 \end{bmatrix}$ $x = -6$
 $y = 4$

7. $x = \#$ of goldfish
 $y = \#$ of bluefish
 $z = \#$ of greenfish



$2.5x + 2y + 1z = 20$ total # to buy fish

$x = 2z$ ratio gold to green fish

$\begin{bmatrix} 2.5 & 2 & 1 & | & 20 \\ 1 & 0 & -2 & | & 0 \end{bmatrix}$ $\xrightarrow{\text{ref}}$ $\begin{bmatrix} 1 & 0 & -2 & | & 0 \\ 0 & 1 & 3 & | & 10 \end{bmatrix}$ $x - 2t = 0$
 $y + 3t = 10$

$(x, y, z) = (2t, 10 - 3t, t)$ $t = \#$ of green fish

$t = 0$	$(0, 10, 0)$	buy 0 gold, 10 blue, and 0 greenfish
$t = 1$	$(2, 7, 1)$	2 7 1
$t = 2$	$(4, 4, 2)$	4 4 2
$t = 3$	$(6, 1, 3)$	6 1 3

Or buy 0, 1, 2 or 3 green fish with the number of gold being twice the number of green and ...

⑧
$$\left[\begin{array}{ccc|c} 1,2 & 0,8 & 1 & 12 \\ -3 & -2 & -2,5 & k \end{array} \right] \xrightarrow{\frac{1}{12}R_1 \rightarrow R_1}$$

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$$\left[\begin{array}{ccc|c} 1 & 2/3 & 5/6 & 10 \\ -3 & -2 & -2,5 & k \end{array} \right] \xrightarrow{3R_1 + R_2 \rightarrow R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 5/6 & 10 \\ 0 & 0 & 0 & k+30 \end{array} \right] \quad \begin{array}{l} x + 2/3 y + 5/6 z = 10 \\ 0 = k + 30 \end{array}$$

$\Rightarrow k = -30$ other values \Rightarrow no soln