Exam 3 Practice Problems

Part 1 –Counting and Probability

(1)

1. Find the probability distribution table for the number of face cards (J, Q, or K) in a hand of 4 cards.

2. A stack of 100 copies has 8 defective copies in it. A sample of 10 is chosen. What is the probability that the sample will have no defective copies?

3. A bowl has 6 green, 7 red and 4 purple jelly beans. A sample of 4 is chosen at random. What is the probability that the sample will have exactly 3 green or exactly one purple jelly bean?

4. Four couples go to the movies. If all 8 people sit down randomly, what is the probability that couples are seated together?

1.

OUTCOME	X	P(X)
0 F.C.	0	$C(12,0)C(40,4)/C(52,4) = 91390/270{,}725 \approx 0.3376$
1 FC	1	$C(12,1)C(40,3)/C(52,4) = 118560/270{,}725 \approx 0.4379$
2FC	2	$C(12,2)C(40,2)/C(52,4) = 51480/270{,}725 \approx 0.1902$
3 FC	3	$C(12,3)C(40,1)/C(52,4) = 8800/270{,}725 \approx 0.0325$
4 FC	4	$C(12,4)C(40,0)/C(52,4) = 495/270{,}725 \approx 0.0018$

2. $\dfrac{C(8,0)C(92,10)}{C(100,10)} = 0.4166$

3. $n(S) = C(17,4) = 2380$

$n(E) = \underbrace{C(6,3)}_{3G} \cdot \underbrace{C(11,1)}_{1G^c} + \underbrace{C(4,1)}_{1P}\, \underbrace{C(13,3)}_{3P^c} - \underbrace{C(6,3)}_{3G} \cdot \underbrace{C(4,1)}_{1P} = 1284$

$P(E) = 1284/2380 \approx 0.5395$

4. $n(S) = 8! = 40320$

$n(E) = 4! \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 384$

$P(E) = \dfrac{384}{40320} \approx 0.0095$

Exam 3 Practice Problems

Part 4 – Binomial and Normal Probability

1. If 1/3 of the students at a very large school are women, what is the probability that in a randomly chosen group of 4 students that there will be at most 1 woman?

binomial. success = woman, $N = 4$, $p = 1/3$
$X = 0, 1$ binomcdf$(4, 1/3, 1) = 0.5926$

2. At a local restaurant 100 people ate bad tuna salad. The probability of getting food poisoning from bad tuna salad is 40%.

(a) What is the probability that fewer then 30 people get sick?
(b) What is the probability that more than 45 people get sick?
(c) What is the probability that between 40 and 50 people get sick?
(d) What is the expected number of sick people? What is the standard deviation in the number of people who get sick?

a) binomial. success = sick, $N = 100$, $p = .4$
$X = 0, 1, \ldots 29$ binomcdf$(100, .4, 29) = 0.0148$

b) $X = 46, 47, \ldots 100$ $1 -$ binomcdf$(100, .4, 45) = 0.1311$

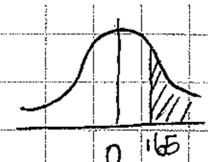
c) $X = 41, 42, \ldots 49$ binomcdf$(100, .4, 49) -$ binomcdf$(100, .4, 40)$
$= 0.4296$

d) $\sigma = 100 \times .4 = 40$ $\sigma = \sqrt{100(.4)(1-.4)} = \sqrt{24} \approx 4.9$

3. The probability that a transistor is defective is 0.2%. A box contains 120 transistors. What is the probability that a box contains at least one defective transistor?

binomial. success = defective. $N = 120$. $p = 0.002$
$X = 1, 2, \ldots 120$ $1 -$ binompdf$(120, .002, 0) = 0.2136$

4. Given that Z is the standard normal variable, find
(a) $P(Z > 0.65)$ (b) $P(Z < 1)$ (c) $P(-1.2 < Z < 0)$ (d) a value of c such that $P(-c<Z<c) = 0.5$

a) 

normalcdf$(.65, 1E99, 0, 1) = 0.2578$

b) normalcdf$(-1E99, 1, 0, 1) = 0.8413$

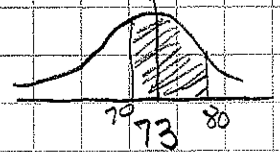
c) normalcdf$(-1.2, 0, 0, 1) = 0.3849$

d) 

$c =$ invNorm$(.75, 0, 1) = .6749$

5. Suppose Math 141 exam 3 scores are normally distributed with a mean of 73 and a standard deviation of 12.

(a) What is the probability that a student earns a C by scoring between 70 and 80?
(b) What is the minimum exam grade required for a student to score in the 90th percentile?
(c) What grades bracket the middle 60% of the students?

a) 

normalcdf (70, 80, 73, 12) = 0.3189

b) 

invNorm (.9, 73, 12) = 88.38

c) 

x1 = invNorm (.2, 73, 12) = 62.90
x2 = invNorm (.8, 73, 12) = 83.10

6. There are 5000 flights per month between Lilliput and the Emerald City. There is a 65% chance that a randomly selected flight will arrive at its destination on time. Use the normal curve approximation to the binomial distribution to estimate the probability that

(a) more than 3300 flights are on time this month
(b) 3200 or fewer flights are on time
(c) between 3220 and 3280 flights are on time

binomial, success = on time, $N = 5000$, $p = .65$

$\mu = 5000(.65) = 3250$ $\quad \sigma = \sqrt{5000(.65)(1-.65)} = \sqrt{1137.5}$
≈ 33.7

a) $X = 3301, \ldots, 5000$, $\quad$ normalcdf(3300.5, 5000.5, μ, σ)
3300.5 $\quad$ 5000.5 $\quad = 0.0672$

b) $X = 0, 1, \ldots, 3200$ $\quad$ normalcdf(-.5, 3200.5, μ, σ) = 0.0711

c) $X = 3221, \ldots, 3279$ $\quad$ normal(3220.5, 3279.5, μ, σ) = .6182

Exam 3 Practice Problems

1

Part 3 – Random Variables and Statistics

1. A sample of jelly bean bags is chosen and the number of blue jelly beans in each bag is counted. The results are shown in the table below:

No. of bags	10	9	8	7	6
No. of blue jelly beans	8	9	10	11	12

(a) What is the expected number of blue jelly beans?
(b) What is the mean, median, mode, and standard deviation in the number of jelly beans?

a) X = # blue jelly beans FR = # of bags ∴ Total bags = 40
$L_1 = 8, 9, 10, 11, 12$ $L_2 = 10/40, 9/40, 8/40, 7/40, 6/40$
$E = 9.75$

b) $\bar{X} = 9.75$, med = 10, mode = 8, $S = 1.4097$ ($\sigma = 1.3919$)

2. A bag contains 10 oranges and 2 of them are rotten. What is the expected number of rotten oranges in a sample of 2?

OUTCOME	X	P(X)
2 rotten	2	$C(2,2)C(8,0)/C(10,2) = 1/45$
1 rotten	1	$C(2,1)C(8,1)/C(10,2) = 16/45$
0 rotten	0	$C(2,0)C(8,2)/C(10,2) = 28/45$

$E(X) = 2(1/45) + 1(16/45) + 0(28/45) = .4$

3. Find the range of values for the random variable X in the following experiments and determine if the random variable is finite discrete, infinite discrete or continuous.
(a) Let X be the number of queens in a hand of 5 cards.
(b) Let X be the time in seconds to swim a 50m race
(c) A bowl has 5 red and 5 green marbles. One marble is chosen at random. If the marble is green, it is replaced in the bowl. Let X be the number of times a marble is chosen until a red marble is picked.

a) $X = 0, 1, 2, 3, 4$ Finite Discrete

b) $X \geq 0$ continuous

c) $X = 1, 2, 3, \ldots$ Inf. Discrete

4. A game is played where a person pays to roll two fair six-sided dice. If exactly one six is shown uppermost, the player wins $5. If exactly 2 sixes are shown uppermost, then the player wins $20. How much should be charged to play this game is the player is to break-even? Round to the nearest cent.

OUTCOME	X	P(X)
one 6	5	10/36
two 6's	20	1/36
no 6's	0	25/36

$E(X) = 5(10/36) + (20)(1/36) + 0(25/36) - \underbrace{C}_{\text{COST TO PLAY}} = 0$

$\Rightarrow C = \$1.94$

5. Mr. Smith buys a $4000 insurance policy on his son's violin. The premium is $50 per year. If the probability that the violin will need to be replaced is 0.8%, what is the insurance company's gain (if any) on this policy?

OUTCOME	X	P(X)
replace	−4000+50	0.008
not repl	50	0.992

$E = (-3950)(.008) + (50)(.992)$

$= \$18$

6. A certain type of battery has an expected useful life of 12 hours with a standard deviation of 2 hours. Use Chebychev's theorem to estimate the following"
   (a) A battery lasts between 9 and 15 hours
   (b) In a batch of 1200 batteries, how many will last more than 18 or fewer than 6 hours?
   (c) Find a value of c such that 8~~4~~% of the batteries last between 12-c hours and 12+c hours.

a) 9 12 15 $3 = k \cdot \sigma = k \cdot 2$ $k = 1.5$ $P(9 \le X \le 15) \geq 1 - 1/1.5^2 = 5/9 = .5556$

b) 6 12 18 $6 = k \cdot 2$ $k = 3$ $P(6 \le X \le 18) \geq 1 - 1/3^2 = 8/9$

$P(X < X \cup X > 18) = 1 - 8/9 = 1/9$

$N = 1200 \times 1/9 = 133.33 \Rightarrow 133$ batteries

c) 12−c 12 12+c $1 - 1/k^2 = .84$ $1/k^2 = .16$ $1/k = .4$

$k = 2.5$ $C = 2 \cdot 2.5 = 5$

7. The odds in favor that a horse will win a race are 3:11. What is the probability the horse will win?

8. The probability of rain is 60%. What are the odds in favor of rain?

7. $P = 3/3+11 = 3/14$

8. $.6/1-.6 = 3/2 \Rightarrow 3:2$ or 3 to 2

9. The following data is the recorded daily high temperature in College Station for March 2006:

83, 81, 77, 74, 77, 83, 80, 82, 79, 85,
86, 86, 75, 72, 69, 77, 72, 69, 76, 76,
65, 58, 51, 61, 69, 74, 72, 67, 73, 81, 82 } L_1

$\mu = 74.58$, med = 76
$\sigma = 8.1430$, mode 69, 72, 77

Find the mean, median, mode and standard deviation for the daily high temperature.

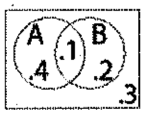
Exam 3 Practice Problems

Part 2 – Conditional Probability

1. Given P(E) = 0.4, P(F) = 0.2 and P(FUF) = 0.5,
   (a) Are E and F independent?
   (b) Are E and F mutually exclusive?

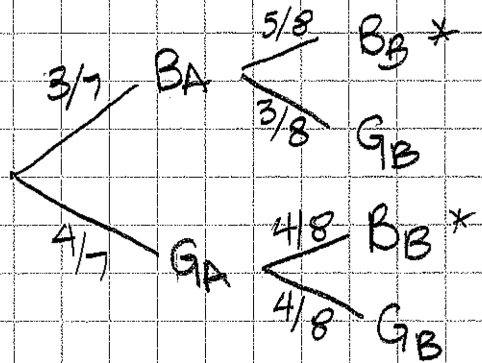
a) $P(E\cup F)=P(E)+P(F)-P(E\cap F) \Rightarrow P(E\cap F)=0.1$
$P(E)\cdot P(F)=(0.4)(.2)=0.08 \neq .1$ not indep
b) $P(E\cap F)\neq 0$ so not M.E.

2. Find $P(B^c|A)$ from the Venn diagram:



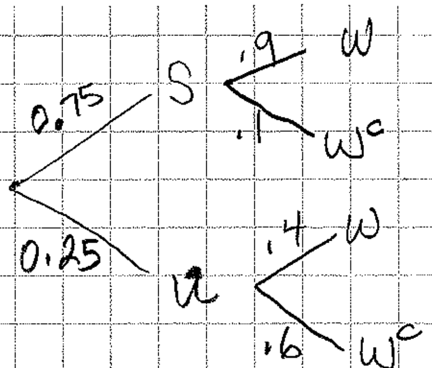
$P(B^c|A)=\dfrac{P(B^c\cap A)}{P(A)}=\dfrac{.4}{.5}$
$=4/5$

3. Urn A has 3 blue and 4 green balls. Urn B has 4 blue and 3 green balls. A ball is chosen from urn A and placed in urn B. A ball is then chosen from urn B. What is the probability that the transferred ball was blue given that the ball drawn from urn B is blue?



$P(B_A|B_B)=P(B_A\cap B_B)/P(B_B)$
$=\dfrac{(3/7)(5/8)}{(3/7)(5/8)+(4/7)(4/8)}$
$=\dfrac{35}{63}=\dfrac{5}{9}\approx 0.5556$

4. A company has rated 75% of its employees as satisfactory and 25% is unsatisfactory. Personnel records indicate that 90% of those rated satisfactory had previous work experience and 40% of those rated unsatisfactory had previous work experience. What is the probability that an employee with previous work experience is unsatisfactory?



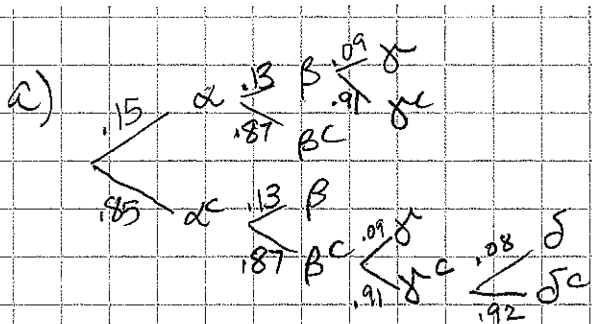
$P(U|W)=P(U\cap W)/P(W)$
$=\dfrac{(.25)(.4)}{(.25)(.4)+(.75)(.9)}$
$=\dfrac{4}{31}\approx 0.1290$

5. An urban area has 4 earthquake faults under it. The table below shows the probability that a particular fault will have a quake of magnitude 6 or greater in the next 20 years.

Fault	Alpha	Beta	Gamma	Delta
probability	15%	13%	9%	8%

(a) What is the probability that none of the faults will have a quake in the next 20 years?
(b) What is the probability that exactly one of the faults will have a quake in the next 20 years?

a)



$P(\alpha^c \cap \beta^c \cap \gamma^c \cap \delta^c) = (.85)(.87)(.91)(.92)$

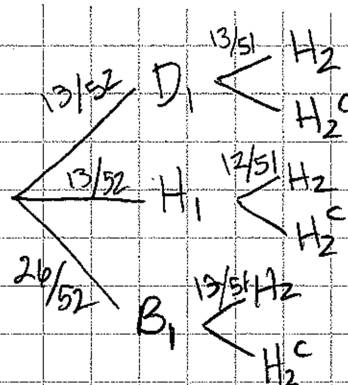
$= 0.6191094$

b) $P = (.15)(.87)(.91)(.92) + (.85)(.13)(.91)(.92) + (.85)(.87)(.09)(.92)$
$+ (.85)(.87)(.91)(.08)$
$= .3168314$

6. Two fair six-sided dice are rolled. Given that the sum shown uppermost is five, what is the probability that a 3 is shown on one of the two dice?

$S = \{1\text{-}4, 2\text{-}3, 3\text{-}2, 4\text{-}1\}$ $\quad P = 2/4$

7. Two cards are chosen in succession from a standard deck of 52 cards. Given that the second card is a heart, what is the probability that the first card was a diamond?



$P(D_1 | H_2) = P(D_1 \cap H_2) / P(H_2)$

$= \dfrac{(13/52)(13/51)}{(13/52)(13/51) + (13/52)(12/51) + (26/52)(13/51)}$

$= \dfrac{13}{51}$