

WEEK 10 REVIEW – Statistics

Example

Determine the possible values for the given random variable and indicate if the random variable is finite discrete, infinite discrete or continuous.

- (a) A hand of 5 cards is dealt from a standard deck of 52 cards. Let X be the number of clubs in the hand of cards.
- (b) A kitten is weighed. Let X be the weight of the kitten in pounds.
- (c) A single card is drawn without replacement from a standard deck of 52 cards. Let X be the number of cards drawn until a red card is picked.
- (d) A single card is drawn with replacement from a standard deck of 52 cards. Let X be the number of cards drawn until a red card is picked.

Example

Use the probability distribution table below to answer the following questions.

X	-2	-1	0	1	2	3
P(X)	0.15	0.05	0.2	0	z	0.25

- (a) Determine the value of z
- (b) Represent this information in a histogram.
- (c) $P(X \leq 0)$
- (d) $P(X > 2)$

The expected value of a random variable X is given by

$$E(X) = x_1 \cdot p_1 + x_2 \cdot p_2 + \dots + x_n \cdot p_n$$

where x_i represents the values that X can have and p_i is the probability that x_i occurs

Example

A raffle sells 500 tickets at \$5 each. There is one first place prize awarded for \$1000, two second place prizes for \$100 each and 10 third place prizes at \$20 each. What is the expected value of a ticket in this raffle?

Example

An Aggie ring is insured for \$1200. The annual premium on the ring insurance is \$15 and the probability that the ring will need to be replaced is 0.5%. What is the insurance company's expected gain on this policy?

Example

One version of roulette has a wheel with 38 equally spaced numbers, 0 – 36 and 00. Half of the numbers from 1 – 36 are red and the other half are black (0 and 00 are green). Calculate the expected value of a \$1 bet with the given payouts.

- (a) Red. Pays \$2.
- (b) First dozen (1 – 12). Pays \$3.
- (c) 00. Pays \$36.

Example

A game consists of choosing two bills at random from a bag that contains two \$20 bills and ten \$1 bills. How much should be charged to play this game to keep it "fair" (expected value of zero)?

Example

A hand of 5 cards is dealt from a standard deck of 52 cards. Let X be the number of clubs in the hand of cards. Find the expected number of clubs

Measures of Central Tendency

The **mean** of the n numbers x_1, x_2, \dots, x_n is $\mu = \frac{x_1 + x_2 + \dots + x_n}{n}$

The **median** of the n numbers x_1, x_2, \dots, x_n is the number in the middle when the n number are arranged in order of size and there are an odd number of values. When there are an even number of values, the median is the mean of the two middle numbers.

The **mode** of the n numbers x_1, x_2, \dots, x_n is the number that occurs the most often. If no number occurs more often than any other number, there is no mode. If two numbers both occur the most often, then there are two modes.

Example

Find the mean, median and mode for the given sets of numbers

- (a) 6, 3, 0, 9, 1, 9, 6, 0, 6, 6, 1, 9, 0, 3, 6, 1, 1, 7, 8, 7, 7, 4, 2, 9, 9
(b) 6, 12, 3, 14, 9, 99

The standard deviation is a measure of spread in a set of n numbers.

Example

Find the standard deviation and variance for the given sets of numbers

- (a) 6, 3, 0, 9, 1, 9, 6, 0, 6, 6, 1, 9, 0, 3, 6, 1, 1, 7, 8, 7, 7, 4, 2, 9, 9
(b) 6, 12, 3, 14, 9, 99

Example

We are given the following data for the number of a certain magazine sold each week at a newsstand during the past year.

# of weeks	5	4	8	11	9	15
# of magazines	15	16	17	18	19	20

- (a) Represent this data in a frequency histogram.
(b) Find the mean, median, mode and standard deviation for this data.

Example

An group 200 apples were weighed and the following results were found. Note that x is the weight of an apple in grams.

Weight (gm)	$130 \leq x < 140$	$140 \leq x < 150$	$150 \leq x < 160$	$160 \leq x < 170$	$170 \leq x < 180$
# apples	10	95	56	33	6

Find the mean and standard deviation for this data.

Example

A sample of grapefruits is selected from a large shipment and the number of seeds in each grapefruit is counted. The following results were found

# grapefruits	6	7	8	9	10
# seeds	7	6	5	4	3

Find the mean, median, mode and standard deviation for this data.

Example

A hand of 5 cards is dealt from a standard deck of 52 cards. Find the standard deviation in the number of clubs in the hand of cards.

The **odds in favor** of an event E occurring is the ratio of $P(E)$ to $P(E^c)$ or $\frac{P(E)}{P(E^c)} = \frac{P(E)}{1 - P(E)}$

Given the odds in favor of an event E are $a:b$, the probability of E is given by $\frac{a}{a+b}$

Example

The odds in favor of selected horses in the 2009 Kentucky Derby were

Backtalk 1:14

Homeboykris 1:60

Sidney's Candy 2:15

- (a) What is the probability that Backtalk will win the race?
- (b) What is the probability that Homeboykris will win the race?
- (c) What is the probability that Sidney's Candy will win the race?

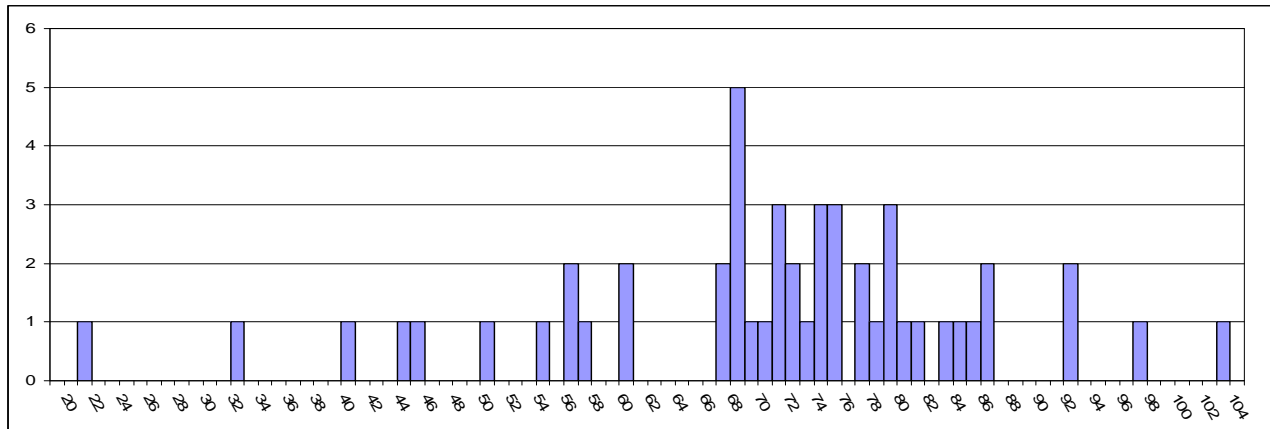
Example

Given that $P(A) = 0.4$, $P(B) = 0.6$, and $P(A \cap B) = 0.3$, find the odds in favor of

- (a) A occurring
- (b) only A occurring

Example

The following exam grades are from a class of 50 students. The mean is 71 and the standard deviation is 16. What is the probability that a randomly selected exam score is within 1 standard deviation of the mean? Within 2 standard deviations of the mean?



To estimate the probability that a data value is within k standard deviations of the mean, use Chebychev's theorem,

$$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

Example

A data distribution has a mean of 100 and a standard deviation of 10. Use Chebychev's theorem to find

- (a) The probability that a data value is between 75 and 125.
- (b) Find a value of c such that the probability that a data value is in the range $100-c$ to $100+c$ is 96%