5

WEEK 3 REVIEW
Multiplication of Matrices

Example
A flower shop sells 96 roses, 250 carnations and 130 daisies in a week. The roses sell for \$2 each, the carnations for \$1 each and the daisies for \$0.50 each. Find the revenue of the shop during the week.

$$R = 96(2) + 250(1) + 130(.5) = 507$$

$$\boxed{\$507}$$

Express the number of flowers in a 1×3 matrix:

$$A = \# \begin{array}{c} \begin{array}{ccc} R & C & D \end{array} \\ \begin{pmatrix} 96 & 250 & 130 \end{pmatrix} \end{array}$$

Next express the price as a 3×1 matrix:

$$B = \begin{array}{c} \\ R \\ C \\ D \end{array} \begin{array}{c} \$ \\ \begin{pmatrix} 2 \\ 1 \\ .5 \end{pmatrix} \end{array}$$

$$A \cdot B = \# \begin{array}{c} \begin{array}{ccc} R & C & D \end{array} \\ \begin{pmatrix} 96 & 250 & 130 \end{pmatrix} \end{array} \cdot \begin{array}{c} \\ R \\ C \\ D \end{array} \begin{array}{c} \$ \\ \begin{pmatrix} 2 \\ 1 \\ .5 \end{pmatrix} \end{array} = \# \begin{array}{c} \$ \\ \begin{bmatrix} 96 \times 2 + 250 \times 1 + 130 \times .5 \end{bmatrix} \end{array}$$

$$= \# \begin{array}{c} \$ \\ \begin{bmatrix} 507 \end{bmatrix} \end{array}$$

$$(1 \times 3) \cdot (3 \times 1)$$

In general, if A is $1\times n$ and B is $n\times 1$, the product AB is a 1×1 matrix:

$$A\cdot B=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n}\end{bmatrix}\cdot\begin{bmatrix} b_{11}\\ b_{12}\\ \vdots \\ b_{n1}\end{bmatrix}=\begin{bmatrix} a_{11}\cdot b_{11}+a_{12}\cdot b_{21}+\dots+a_{1n}\cdot b_{n1}\end{bmatrix}$$

If A is an $m\times n$ matrix and B is a $n\times p$ matrix, then the product matrix $A\cdot B=C$ is an $m\times p$ matrix.

$$A\cdot B=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{m1} & a_{m2} & & a_{mn}\end{bmatrix}\cdot\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p}\\ b_{21} & b_{22} & \dots & b_{2p}\\ \vdots & \vdots & \ddots & \vdots\\ b_{n1} & b_{n2} & \dots & b_{np}\end{bmatrix}$$

row 1 col 2

$$=\begin{bmatrix} a_{11}\cdot b_{11}+a_{12}\cdot b_{21}+\dots+a_{1n}\cdot b_{n1} & (ab)_{12} & \dots & (ab)_{1p}\\ (ab)_{21} & (ab)_{22} & \dots & (ab)_{2p}\\ \vdots & \vdots & \ddots & \vdots\\ (ab)_{m1} & (ab)_{m2} & \dots & (ab)_{mp}\end{bmatrix}$$

row 2 col 1

Matrix multiplication is not commutative. In general, $AB\neq BA$

Example
Find the products AB and BA where

$$A=\begin{bmatrix} 1 & 0\\ -2 & 3\end{bmatrix} \text{ and } B=\begin{bmatrix} -1 & 2\\ 0 & -3\end{bmatrix}$$

$$A\cdot B=\begin{bmatrix} 1 & 0\\ -2 & 3\end{bmatrix}\begin{bmatrix} -1 & 2\\ 0 & -3\end{bmatrix} = \begin{bmatrix} (1)(-1)+0(0) & (1)(2)+(0)(-3)\\ (-2)(-1)+3(0) & (-2)(2)+(3)(-3)\end{bmatrix}$$

$$(2\times 2)\cdot(2\times 2) = 2\times 2$$

$$=\begin{bmatrix} -1 & 2\\ 2 & -13\end{bmatrix}$$

$$B\cdot A=\begin{bmatrix} -1 & 2\\ 0 & -3\end{bmatrix}\begin{bmatrix} 1 & 0\\ -2 & 3\end{bmatrix}=\begin{bmatrix} \_\_ & \_\_\\ \_\_ & \_\_\end{bmatrix}=\begin{bmatrix} -5 & 6\\ 6 & -9\end{bmatrix}$$

One special matrix is called the identity matrix, I.
It is a square matrix with 1's on the diagonal and zeros elsewhere,

$I_n \to$ nxn identity matrix

$$I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

The identity matrix has the following properties:

$$AI = A = IA$$

Matrix multiplication and linear equations:

Example: Write the following ~~system~~ of linear equations as a matrix equation

$$A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad B = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$$2x - 3y = 6$$
$$-x + 2y = 4$$

$$AX = \underset{(2\times 2)}{\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}} \cdot \underset{(2\times 1)}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{2\times 1}{\begin{pmatrix} 2x - 3y \\ -x + 2y \end{pmatrix}} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} = B$$

$$AX = B$$

Example: Cost Analysis - The Mundo Candy Company makes three types of chocolate candy: cheery cherry (cc), mucho mocha (mm) and almond delight (ad). The candy is produced in San Diego (SD), Mexico City (MC) and Managua (Ma) using two main ingredients, sugar (S) and chocolate (C).

Each kg of cheery cherry requires 0.5 kg of sugar and 0.2 kg of chocolate.
Each kg of mucho mocha requires 0.4 kg of sugar and 0.3 kg of chocolate.
Each kg of almond d. requires 0.3 kg of sugar and 0.3 kg of chocolate.

(a) Put this information in a 2x3 matrix.

$$\begin{array}{c c} & \begin{array}{ccc} cc & mm & ad \end{array} \\ \begin{array}{c} Su \\ ch \end{array} & \begin{pmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{pmatrix} \end{array}$$

(b) The cost of 1 kg of sugar is \$3 in San Diego, \$2 in Mexico City and \$1 in Managua. The cost of 1 kg of chocolate is \$3 in San Diego, \$3 in Mexico City and \$4 in Managua.

Put this information into a matrix in such a way that when it is multiplied by the matrix in part (a) it will tell us the cost of producing each kind of candy in each city. ⇒ 3x3 matrix

~~B_1 is 2x3~~ ~~or~~ B_2 is 3x2

AB_1 (2x3)(2x3) NO

or B_1A (2x3)(2x3) NO

or AB_2 (2x3)(3x2) = 2x2 NO

or B_2A (3x2)·(2x3) = 3x3 Yes?

$$\begin{array}{c c} & \begin{array}{cc} S & C \end{array} \\ \begin{array}{c} \text{cost in SD} \\ \text{cost in MC} \\ \text{cost in Ma} \end{array} & \begin{pmatrix} 3 & 3 \\ 2 & 3 \\ 1 & 4 \end{pmatrix} \end{array} \cdot \begin{array}{c c} & \begin{array}{ccc} cc & mm & ad \end{array} \\ \begin{array}{c} S \\ C \end{array} & \begin{pmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{pmatrix} \end{array} = \begin{array}{c c} & \begin{array}{ccc} cc & mm & ad \end{array} \\ \begin{array}{c} SD \\ MC \\ Ma \end{array} & \begin{pmatrix} 3x.5+3x.2 & & \\ & & \\ & & \end{pmatrix} \end{array}$$

3x3