WEEK 7 REVIEW: SETS and COUNTING

set builder notation

Example

Let $U = \{x \mid x$ is a postive integer less than $8\}$,
$A = \{1,2,3,4\}$, $B = \{3,4,5\}$, and $C = \{5,6,7\}$

a. Write U in roster notation $U = \{1,2,3,4,5,6,7\}$

b. $A^c =$ $\{5,6,7\}$

If a set has n elements then there are 2^n subsets

c. $A \cap B =$ $\{3,4\}$ intersect AND

d. $A \cup B =$ $\{1,2,3,4,5\}$ Union "OR"

e. List all the subsets of C $n(C) = 3 \Rightarrow 2^3 = 8$ subsets

$\emptyset, \{5\}, \{6\}, \{7\}, \{5,6\}, \{5,7\}, \{6,7\}, \{5,6,7\}$

proper subsets

Determine if the statements below are true or false

f. A and C are disjoint sets TRUE

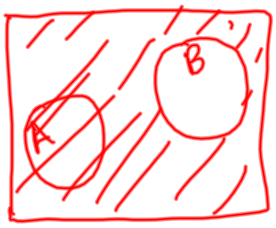
$\emptyset \subseteq A$ $\emptyset \subset A$ $1 \in A$ TRUE

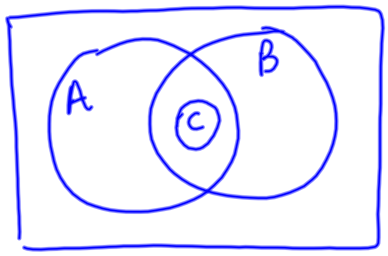
g. $\emptyset \in A$ false

h. $\{5,6,7\} \subset C$ false $\{5,6,7\} \subseteq C$

Example: Use Venn diagrams to indicate

a. $A \subset U, B \subset U, A \subset B^c$ b. $A \subset U, B \subset U, C \subset U, C \subset A \cap B$

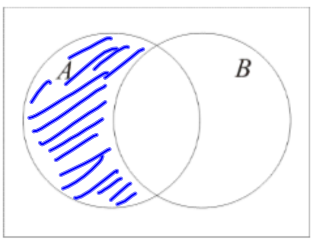




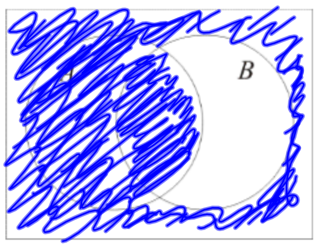
Example

Shade the indicated regions on the Venn diagram

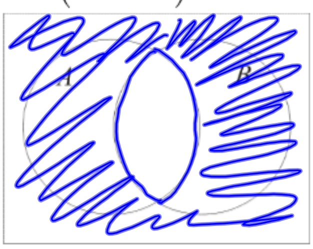
a. $A \cap B^c$



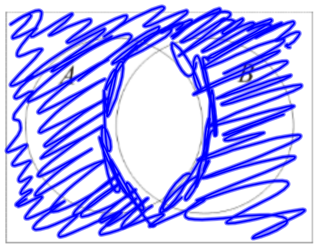
b. $A \cup B^c$



c. $(A \cap B)^c$

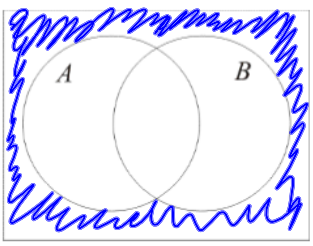


d. $A^c \cup B^c$

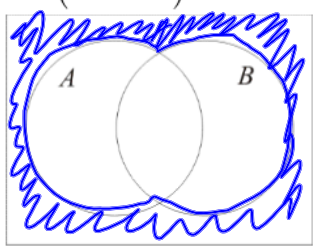


de Morgan's Laws

e. $A^c \cap B^c$

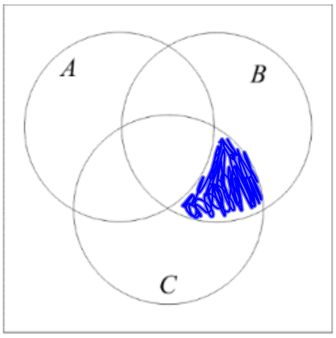


f. $(A \cup B)^c$

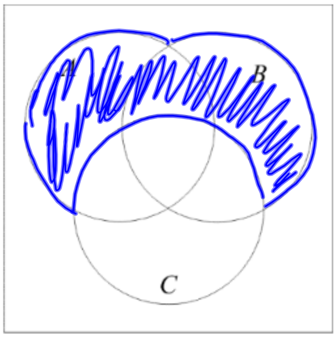


de Morgan's Laws

g. $A^c \cap B \cap C$

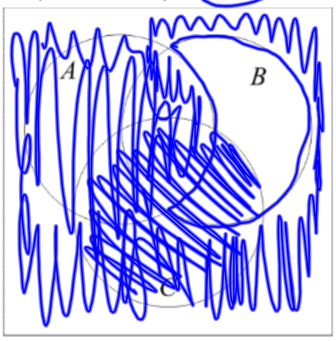


h. $(A \cup B) \cap C^c$

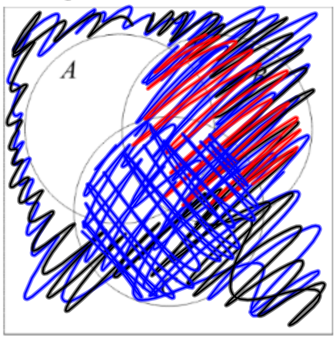


all of C

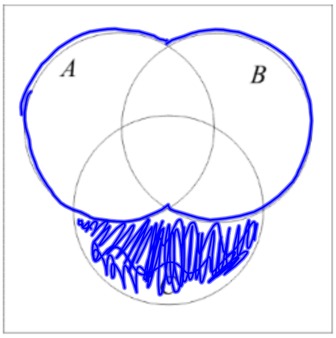
i. $(A^c \cap B)^c \cup C$



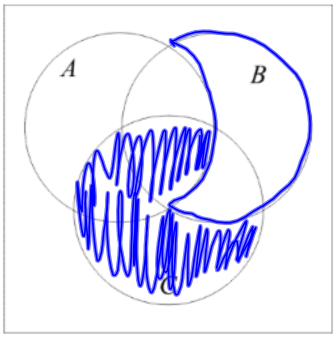
j. $A^c \cup B \cup C$



k. $(A \cup B)^c \cap C$



l. $(A^c \cap B)^c \cap C$



Example

Let U be the set of all staff at Texas A&M University and let

$A = \{x | x \text{ owns an automobile}\}$

$H = \{x | x \text{ owns a house}\}$

$P = \{x | x \text{ owns a piano}\}$

Describe the following sets in words

a. A^c A staff member who does not own an automobile

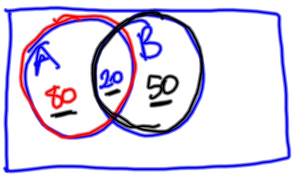
b. $A \cap H^c$ A staff member who owns and auto and does not own a house

c. $A^c \cup P^c$ A staff member who does not own an auto or doesnot own a piano

d. $A^c \cap H^c \cap P^c$ A staff member who does not own an auto and does not own a house and does not own a piano

Example

If $n(A) = 100$, $n(A \cap B) = 20$, and $n(A \cup B) = 150$, what is $n(B)$?

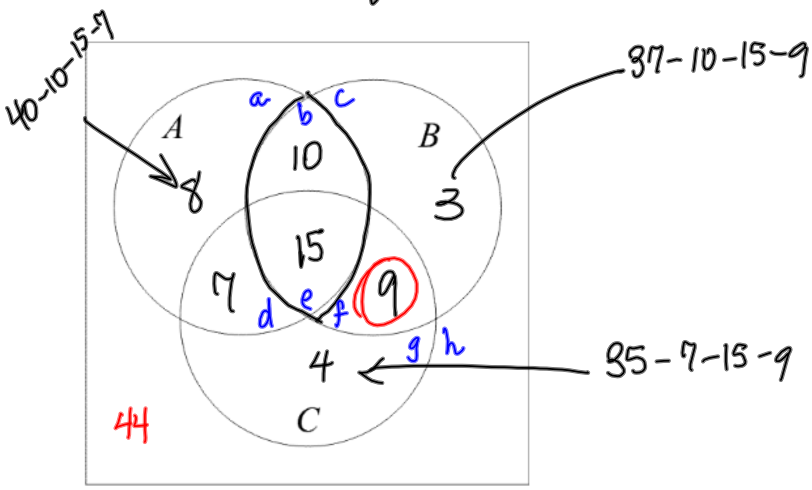


150- 80 -20=50

$n(B) = 70$

Example

Given $n(U)=100$, $n(A)=40$, $n(B)=37$, $n(C)=35$, $\boxed{n(A\cap B)=25}$, $n(A\cap C)=22$, $n(B\cap C)=24$, and $n(A\cap B\cap C^c)=10$, find $n(A^c\cap B\cap C)$. $=9$



$n(A)=a+b+d+e=40$

$n(A\cap C)=d+e=22$

etc

$120=x+y+z+w$ // $55=x+y$, $40=y+z$, $70=x+y+z$

Example

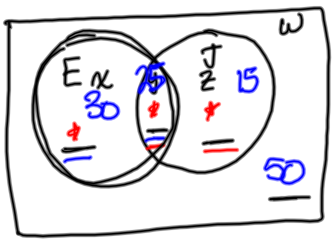
In a survey of 120 adults, 55 said they had an egg for breakfast that morning, 40 said they had juice for breakfast, and 70 said they had an egg or juice for breakfast. How many had an egg but no juice for breakfast? How many had neither an egg nor juice for breakfast?

30

50

$E=\{x \mid x$ is a person who had an egg$\}$

$J=\{x \mid x$ ..... juice$\}$



$n(E\cup J)=x+y+z=(x+y)+(y+z)-y$

$\boxed{n(E\cup J)=n(E)+n(J)-n(E\cap J)}$ UNION RULE

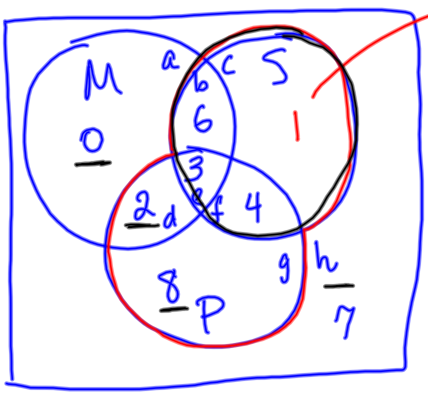
$70=55+40-n(E\cap J) \rightsquigarrow n(E\cap J)=25$

Example

Determine how many pizzas were sold if

- 3 pizzas had mushrooms, pepperoni, and sausage e
- 7 pizzas had pepperoni and sausage $e+f$
- 6 pizzas had mushrooms and sausage but not pepperoni b
- 15 pizzas had two or more of these toppings $b+d+e+f$
- 11 pizzas had mushrooms $a+b+d+e$
- 8 pizzas had only pepperoni g
- 24 pizzas had sausage or pepperoni $b+c+d+e+f+g$
- 17 pizzas did not have sausage $a+d+g+h$

$24-6-3-4-2-8$

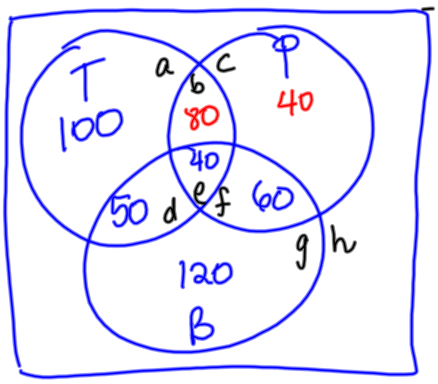


Example

Six hundred people were surveyed and it was found that during the past year,

- 330 did not travel by bus, $a+b+c+h$
- 100 traveled by plane but not by train, $c+f$ $\quad 100 = c + \cancel{f}^{60} \Rightarrow c = 40$
- 150 traveled by train but not by plane, $a+d$
- 120 traveled by bus but not by train or plane, g
- 100 traveled by both bus and plane, $\cancel{e}^{40}+f = 100$
- 40 traveled by all three, and e
- 220 traveled by plane. $b+\cancel{c}^{40}+\cancel{e}^{40}+\cancel{f}^{60} = 220 \Rightarrow b = 80$

How many did not travel by any of these three modes of transportation?



$600 = a+\cancel{b}^{80}+\cancel{c}^{40}+d+\cancel{e}^{40}+\cancel{f}^{60}+\cancel{g}^{120}+h$

$300 = a+\cancel{b}^{80}+\cancel{c}^{40}+h$

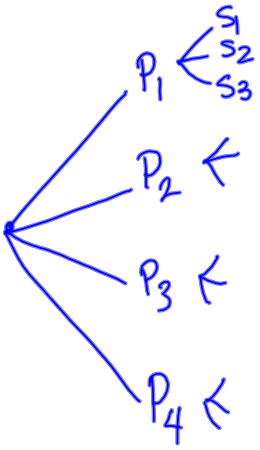
$150 = a+d$

$260 = a+d+h \Rightarrow h = 110$

$210 = a+h$

$150 = a+d$

Example
At a pasta diner there is a choice of 4 different pastas and 3 different sauces. How many dinners can be made?



$S = \{P_1S_1, P_1S_2, P_1S_3, P_2S_1, \ldots P_4S_3\}$

$\# \text{ of dinners} = \underset{\text{pasta}}{\underline{4}} \cdot \underset{\text{sauce}}{\underline{3}}$

multip princip

Example: How many different 4-digit access codes can be made if 0000, 0001, ... 9999

a. there are no restrictions? $\underset{1^{st}}{\underline{10}} \cdot \underset{2^{nd}}{\underline{10}} \cdot \underset{3^{rd}}{\underline{10}} \cdot \underset{4^{th}}{\underline{10}} = 10{,}000$

b. there are no repeats? $\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7}$

c. the first digit cannot be a 0 or a 1 and no repeats are allowed? $\underline{8} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7}$

d. four of the same digit is not allowed?

$\underline{10}\ \underline{10}\ \underline{10}\ \underline{10} - \underset{\text{not ok}}{10} = 9990$

1111 NO
1112, 1121, 1211, 2111 are ok

NOT OK:
0000, 1111, 2222, ... 9999
10

Example

A minivan can hold 7 passengers. An adult must sit in one of the two front seats and anyone can sit in the rear 5 seats. A group of 4 adults and 3 children are to be seated in the van. How many different seating arrangements are possible?

~~A~~AA~~A~~ ooo

$$\underset{F1}{\underline{4}} \cdot \underset{F2}{\underline{3}} \cdot \underset{R1}{\underline{5}} \cdot \underset{R2}{\underline{4}} \cdot \underset{R3}{\underline{3}} \cdot \underset{R4}{\underline{2}} \cdot \underset{R5}{\underline{1}} = 1440$$

~~B~~BBB~~B~~BGG~~G~~GG~~G~~

Example

You have a class of 12 children, 6 boys and 6 girls. How many ways can the children be seated in a row

a. if boys and girls must alternate?

$$\underset{B \text{ or } G}{\underline{12}} \cdot \underline{6} \cdot \underline{5} \cdot \underline{5} \cdot \underline{4} \cdot \underline{4} \cdot \underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1} = 1{,}036{,}800$$

$$= 2 \cdot 6!\,6!$$

b. if a girl must be seated at each end?

$$\underset{G1}{\underline{6}} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underset{G2}{\underline{5}} =$$

c. if all the boys sit together and all the girls sit together?

$$\underbrace{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{\text{BOYS}} \, \underbrace{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{\text{GIRLS}} \; \underset{\text{or}}{+} \; \underbrace{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{\text{GIRLS}} \, \underbrace{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{\text{BOYS}}$$

$$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1 \quad \text{and} \quad 0! = 1$$

6 math → PRB → 4: !

Example
You take a multiple choice test with 3 questions and each question has 5 possible answers. How many ways can the test be answered?

$$\underset{Q1}{\underline{5}} \cdot \underset{Q2}{\underline{5}} \cdot \underset{Q3}{\underline{5}} = 125$$

Example
MJ

Matthew and Jennifer go to the movies with four of their friends. How many ways can these six children be seated if

a. there are no restrictions? $\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 720$

$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underset{\text{MJ or JM}}{\underline{2}} = 240$

b. Matthew and Jennifer are seated next to each other?

c. Matthew and Jennifer are not next to each other?

$$\underset{\text{no restr}}{\underline{720}} - \underset{\text{are together}}{\underline{240}} = 480$$

Example
Four couples are going to the movie together. How many ways can these eight people be seated if couples sit together?

AB CD EF GH

$3 \cdot 2 \cdot 1 = 3!$

$$\underset{1^{st}C}{\underline{4}} \cdot \underset{C2}{\underline{3}} \cdot \underset{C3}{\underline{2}} \cdot \underset{C4}{\underline{1}} \cdot \underset{\text{AB or BA}}{\underline{2!}} \cdot \underset{\text{CD DC}}{\underline{2}} \cdot \underline{2} \cdot \underline{\cancel{2}}$$

Combinations

Example: How many ways can we choose a *group* of 4 students from a class of 10 students?

$$\frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210 = C(10,4) = 10\,nCr\,4$$

This is the number of *combinations* of 10 items taken 4 at a time.

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

Example: A lottery has 54 numbers and 6 are chosen to be winning numbers. If the order the numbers are picked doesn't matter, how many ways can these numbers be chosen?

$C(54,6) = 25,827,165$

6 winning and 48 losing numbers

How many ways to choose no winning numbers?

$\underbrace{C(6,0)}_{0\text{ win}} \cdot \underbrace{C(48,6)}_{6\text{ losing}} = 12,271,512$

How many ways to choose at least 3 winning numbers?

$\underbrace{\underbrace{C(6,3)}_{3W} \text{ and } \underbrace{C(48,3)}_{3L}}_{345,920} + (\text{OR})\ \underbrace{C(6,4)}_{4W} \cdot \underbrace{C(48,2)}_{\text{and } 2L} + (\text{OR})\ \underbrace{C(6,5)}_{5W} \cdot \underbrace{C(48,1)}_{\text{and } 1L} + (\text{OR})\ \underbrace{C(6,6)}_{6W} \cdot \underbrace{C(48,0)}_{\text{and } 0L}$

$345,920 + 16,920 + 288 + 1$

Example: From 10 names on a ballot, how many ways can 4 be elected to a committee if ~~each~~ one person is the chair and another is the vice-chair?

$$\underset{Ch}{10} \cdot \underset{VC}{9} \cdot \underset{2\text{ members}}{C(8,2)} = 2520 = \underset{\text{Pick com}}{C(10,4)} \cdot \underset{Ch}{4} \cdot \underset{VC}{3}$$

$$\#\text{ways} = \frac{N!}{n_1!\, n_2! \cdots n_k!}$$

Example: How many ways can 10 books be arranged on a shelf if there are 3 identical biology books, 6 identical physics books and a math book?

$$\frac{10!}{\underbrace{3!}_{Bio}\,\underbrace{6!}_{Phy}\,\underbrace{1!}_{M}} = 840$$

Example: You have a group of 13 different books. Three are math books, four are chemistry and six are history books. How many different arrangements are possible if books of the same type are kept together?

$M_1 M_2 M_3$ $C_1 C_2 C_3 C_4$ $H_1 H_2 H_3 H_4 H_5 H_6$

$$\underset{\text{arr the type of book}}{3 \cdot 2 \cdot 1} \cdot \underset{\substack{\text{math} \\ 3\cdot2\cdot1}}{3!} \cdot \underset{\substack{\text{Chem} \\ 4\cdot3\cdot2\cdot1}}{4!} \cdot \underset{\substack{\text{Hist} \\ 6\cdot5\cdot4\cdot3\cdot2\cdot1}}{6!}$$

Example: How many ways can 6 girls and 3 boys be arranged in a row if boys cannot be next to each other?

$\_\ G_1\ \underline{B_1}\ G_2\ \underline{B_2}\ G_3\ \_\ G_4\ \_\ G_5\ \underline{B_3}\ G_6\ \_$

$$\underset{\text{arr } G}{6!} \quad \underset{\text{spot for boys}}{C(7,3)} \quad \underset{\text{arr } B}{3!}$$