WEEK 10 REVIEW (7.2 – 7.4)

> A sample space in which each of the outcomes has the same chance of occurring is called a ***uniform sample space***.

The ***probability*** of event E is $P(E)$, a number between 0 and 1. It is the ratio of the number of outcomes in event E, $n(E)$ to the number of outcomes in the sample space, $n(S)$:

$$P(E) = \frac{n(E)}{n(S)}$$

The ***union rule*** for sets can be applied to probability:

$$\underbrace{n(E \cup F)}_{n(S)} = \underbrace{n(E)}_{n(S)} + \underbrace{n(F)}_{n(S)} - \underbrace{n(E \cap F)}_{n(S)} \text{ becomes}$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

We can also find the empirical probability of an event by finding the ***relative frequency*** of the event.

A ***probability distribution table*** has the following properties:

1. Each of the entries is mutually exclusive with all other entries
2. The sum of the probabilities is 1

Events that can't occur at the same time are called ***mutually exclusive***. Note that the simple events are mutually exclusive.

A ***simple event*** contains exactly one outcome

$S_{BAD} = \{W, O, D\}$

Example: A letter is chosen at random from the word WOOD. How many outcomes are in the uniform sample space for this experiment?

$\underbrace{S = \{W, O, O, D\}}_{\text{set of all outcomes uniform}}$, $n(S) = 4$, Simple events are $\{W\}, \{O\}, \{O\}, \{D\}$

Example: A bowl has 3 blues ($B_1 B_2 B_3$) and 2 red ($R_1 R_2$) beads in it. Two beads are chosen at random from the bowl. How many outcomes are in the uniform sample space for this experiment?

$B_1B_2, B_1B_3, B_1R_1, B_1R_2, B_2B_3, B_2R_1, B_2R_2, B_3R_1, B_3R_2, R_1R_2$

$C(5,2) = 10$

Example: Suppose that $S = \{a, b, c\}$ is a uniform sample space. If $E = \{a, b\}$ and $F = \{b, c\}$, what is $P(E \cap F)$? $E \cap F = \{b\}$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{3}$$

Example: At a swim meet, 20% of the girls finished a race in less than 25 seconds. 55% of the girls finished this same race in 30 or fewer seconds. 15% of the girls took 35 seconds or more to finish this race. Arrange this information into a probability distribution * * table.

t = time in seconds to swim the race

Event	Prob
$t < 25$	0.20
$25 \le t \le 30$	0.35
$30 < t < 35$	0.30
$t \geq 35$	0.15

$.55$

$1 - .2 - .35 - .15$

event	P
$t < 25$	0.2
$t \le 30$	.55
$t \geq 35$	.15

not MUT. EXCL

TRUE BUT NOT A PROB DIST TABLE

$\frac{6}{1st} \cdot \frac{6}{2nd} = 36 = n(S)$

Example: Two fair six-sided dice are rolled. What is the probability that the sum of the numbers shown uppermost is 9 or at least one three is showing uppermost?

UNION

E F

1-1	2-1	3-1	4-1	5-1	6-1
1-2	2-2	3-2	4-2	5-2	6-2
1-3	2-3	3-3	4-3	5-3	6-3
1-4	2-4	3-4	4-4	5-4	6-4
1-5	2-5	3-5	4-5	5-5	6-5
1-6	2-6	3-6	4-6	5-6	6-6

F E

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{4}{36} + \frac{11}{36} - \frac{2}{36} = \frac{13}{36}$

Example: Suppose we have a jar with 8 blue and 6 green marbles. Find the probability distribution table for the number of blue marbles in the sample of 2 marbles and find the probability there is at least one blue marble.

7.4

$n(S) = C(14,2) = 91$

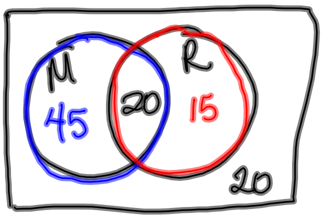
EVENT	PROB
0B 2G	$\frac{C(8,0)\cdot C(6,2)}{C(14,2)} = \frac{15}{91}$
1B 1G	$\frac{C(8,1)\, C(6,1)}{C(14,2)} = \frac{48}{91}$
2B 0G	$\frac{C(8,2)\, C(6,0)}{C(14,2)} = \frac{28}{91}$

at least one blue

$\frac{48}{91} + \frac{28}{91} = \frac{76}{91}$

Example: When shopping for a ceiling fan, you find after looking at 100 fans that 65 of the fans have more than 4 blades, 35 of the fans are reversible and 80 of the fans have more than 4 blades or are reversible. What is the probability ~~that a reversible~~ fan has more than 4 blades? and is reversable

$P(M \cap R) = \frac{20}{100}$



$n(S) = 100$

$n(M) = 65$

$n(R) = 35$ ✓

$n(M \cup R) = 80 = n(M) + n(R) - n(M \cap R)$

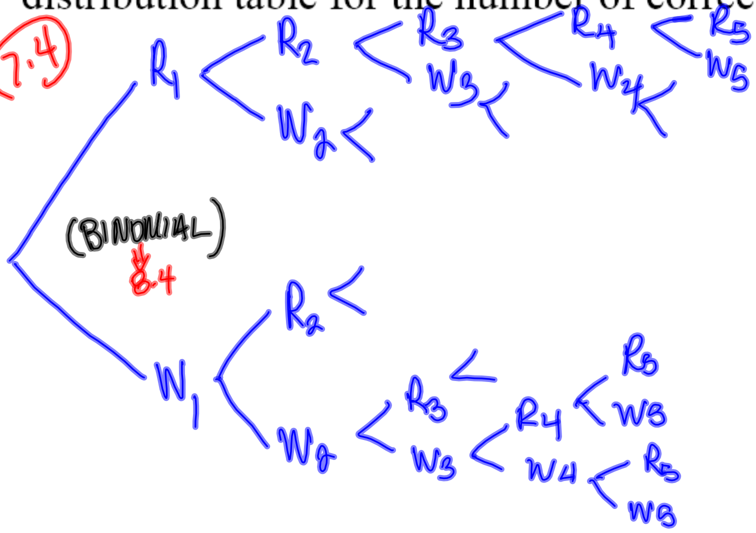
$80 = 65 + 35 - n(M \cap R)$

$\rightarrow n(M \cap R) = 20$

$n(S) = \underset{1st}{2} \cdot \underset{2nd}{2} \cdot \underset{3rd}{2} \cdot \underset{4th}{2} \cdot \underset{5th}{2} = 32$

Example: A student takes a true/false test with 5 questions by guessing (choose answer at random). Write a probability distribution table for the number of correct answers.

(7.4)



EVENT	PROB
5R	$\frac{C(5,5)}{32} = \frac{1}{32}$
4R	$\frac{C(5,4)}{32} = \frac{5}{32}$
3R	$C(5,3)/32 = 10/32$
2R	$C(5,2)/32 = 10/32$
1R	$C(5,1)/32 = 5/32$
0R	$C(5,0)/32 = 1/32$

$S = \{R_1R_2R_3R_4R_5, R_1R_2R_3R_4W_5, \ldots\ldots, W_1W_2W_3W_4W_5\}$

Example: You are dealt 3 cards from a standard deck of 52 cards. Find the probability distribution table for the number of spades in your hand of 3 cards.

$n(S) = C(52,3) = 22{,}100$

EVENT	PROB
0S 3S^c	$\frac{C(13,0)\,C(39,3)}{C(52,3)} = \frac{9139}{22{,}100} \approx 41\%$
1S 2S^c	$\frac{C(13,1)\,C(39,2)}{C(52,3)} = \frac{9633}{22{,}100} \approx 44\%$
2S 1S^c	$\frac{C(13,2)\,C(39,1)}{C(52,3)} = \frac{3042}{22100} \approx 14\%$
3S 0S^c	$\frac{C(13,3)\,C(39,0)}{C(52,3)} = \frac{286}{22100} \approx 1\%$

Example: There are 72 marbles in a box. There are 18 different colors and 4 marbles of each color. Five marbles are chosen at random from the box. What is the probability of a full house? That is, 3 of one color and 2 of a different color.

$n(S) = C(72,5) = 13{,}991{,}544$

$$n(E) = \underbrace{18}_{\text{Pick a color}} \cdot \underbrace{C(4,3)}_{\text{Pick 3}} \cdot \underbrace{17}_{\text{Pick a 2nd color}} \cdot \underbrace{C(4,2)}_{\text{Pick 2}} = 7344$$

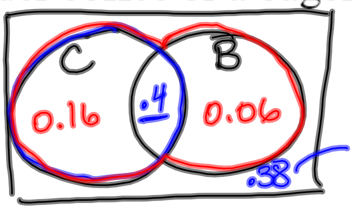
$$P(E) = \frac{7344}{13{,}991{,}544}$$

6/100

$$\boxed{P(E) + P(E^c) = 1}$$

Example: A coffee shop finds that 44% of its customers do not order coffee, ~~16% order only coffee~~ and ~~6% order only a bagel~~. What is the probability that a randomly selected customer will order coffee or a bagel?

$P(C^c) = .44 \Rightarrow P(C) = 1 - .44 = .56$



$1 - .16 - .4 - .06$

$P(C \cup B) = .16 + .4 + .06 = .62$

$= 62\%$

Example: A box has 30 transistors and a sample of 5 is chosen for testing to decide if the box is "good" or "bad". A box is considered "bad" if one or more transistors in the sample are found to be defective. What is the probability that a box that has 4 defective transistors will be considered "good"?

(26 GOOD)

$$P = \frac{C(\overset{\text{GOOD}}{26},5)\cdot(\overset{\text{BAD}}{4},0)}{C(30,5)} = \frac{65780}{142506} \quad (\approx 46\% \;!!)$$

Example: Matthew is studying for a Latin quiz and he learns the meaning of 24 nouns from the list of 30. The Latin quiz has 10 nouns. If a passing grade is 7 or more, what is the probability that Matthew passes this Latin quiz?

6 does not know

$$P = P(7) + P(8) + P(9) + P(10)$$

$$= \frac{C(\overset{\text{KNOW}}{24},7)C(\overset{\text{DK}}{6},3) + C(24,8)C(6,2) + C(24,9)C(6,1) + C(24,10)C(6,0)}{C(30,10)}$$

$$= \frac{27{,}760{,}425}{30{,}045{,}015} \quad (\approx 92\%)$$

12 coins small

Example: A bowl has 3 pennies, 5 nickels and 4 quarters. Four coins are selected at random from the bowl. What is the probability that exactly 3 nickels or exactly one quarter is chosen?

E U F

$\boxed{n(S) = C(12,4) = 495}$

$n(E) = \underset{3N \cdot 1N^c}{C(5,3)\cdot C(7,1)} = 70$ task

$n(F) = \underset{1Q \cdot 3Q^c}{C(4,1)\cdot C(8,3)} = 224$ task

$n(E \cap F) = \underset{3N \cdot 1Q \cdot 0P}{C(5,3)\cdot C(4,1)\cdot C(3,0)} = 40$

$$P(E \cup F) = \frac{70 + 224 - 40}{495} = \frac{254}{495}$$

$$= \frac{\overbrace{C(5,3)C(7,1)}^{3N\,1N^c} + \overbrace{C(4,1)C(8,3)}^{1Q\,3Q^c} + \overbrace{C(5,3)C(4,1)C(3,0)}^{3N\,1Q}}{C(12,4)}$$