WEEK 14B REVIEW (5.1 – 5.3)

Section 5.1 Compound Interest

Simple interest – interest computed as a percentage of the principal

Simple interest earned: $I = Prt$

I=interest earned
P=principal
r=annual interest rate (decimal form)
* t=time period (in years) *

Future value: $F = P + I = P + Prt = P(1 + rt)$

Example: If a bank loans \$525 to an individual for 3½ years at 7.25% simple interest, what will be the amount repaid on the loan?

$I = 525 * (.0725)(3.5) = 133.22$

Repay $525 + 133.22 = \$658.22$

Example: If a \$1000 deposit grows in value to \$1024 after 9 months, what is the simple interest rate that is earned?

$I = 1024 - 1000 = 24$

$24 = P \cdot r \cdot t = 1000 \cdot r \cdot (9/12)$

$r = \frac{24}{1000(3/4)} = 0.032 \Rightarrow 3.2\%$

Example

How much should be placed in an account that pays simple interest of 4% so that the value of the account after 18 months is $3000?

$3000 = P(1 + (.04)(1.5)) = 1.06P$

$P = \frac{3000}{1.06} = \$2830.19$

Example

You have an account with $500 that pays 5% simple interest. How long until your account doubles in value?

$2 \times 500 = 1000 = 500(1 + 0.05t)$ ÷ 500

$2 = 1 + 0.05t \Rightarrow 1 = .05t \Rightarrow t = \frac{1}{.05} = 20$

20 years

When interest is paid periodically and the interest earns interest, we have compound interest.

Example

You deposit $100 into an account that pays 10% annual interest that is compounded annually. How much is in the account after 4 years?

End of yr 1 : $F_1 = 100(1 + .1 \times 1) = 100 \times 1.1 = 110$

End of yr 2 : $F_2 = 110(1 + .1 \times 1) = 110 \times 1.1 = 100 \times 1.1 \times 1.1 = 100(1.1)^2 = 121$

End of yr 3 : $F_3 = 121(1 + .1 \times 1) = 121 \times 1.1 = 100 \times 1.1^3 = 133.(10)$

End of yr 4 : $= 100(1.1)^4 = 146.41$

$\Rightarrow$ $6.41 more than simple interest

ON THE CALCULATOR, TVM Solver:

be careful

N is #of compounding periods

I% is the interest rate AS A PERCENT

PV is Present Value

PMT is Payments

FV is future value

$ MUST HAVE A SIGN CHANGE

P/Y and C/Y is Payments/cmp periods per year (= m)

set PMT: END (make payments at the end of the cycle)

If interest is compounded	Then m is
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

not exact

Example: You deposit \$100 into an account that pays 10% annual interest that is compounded annually. How much is in the account after 4 years?

N = 4x1 = 4

I = 10

PV = 100

PMT = 0

FV = ? ⇒ $\boxed{\$146.41}$

P/Y = 1

Example: You deposit $300 into an account that pays 6% annual interest compounded monthly. How much is in the account after 3 months?

N = 3
I = 6
PV = 300
PMT = 0
FV = ?
P/Y = 12
→ $304.52

Example: You deposit $1000 into an account that pays 10% annual interest. How much is in the account after 10 years if

(a) the account earns simple interest?

$F = 1000(1 + .1 \times 10) = 2000$

(b) the account earns interest compounded annually?

N = 10×1 = 10
I = 10
PV = 1000
PMT = 0
FV = ?
P/Y = 1
→ $2593.74

(c) the account earns interest compounded semiannually?

N = 10×2 = 20
⋮
P/Y = 2
→ $2653.30 (incr of $59.56)

(d) the account earns interest compounded quarterly?

N = 10×4 = 40
P/Y = 4
→ $2685.06 (incr of $31.76)

(e) the account earns interest compounded monthly?

N = 10×12 = 120
P/Y = 12
→ $2707.04 (incr of $21.98)

(f) the account earns interest compounded weekly?

N = 10×52 = 520
P/Y = 52
$2715.67 (incr of $8.63)

(g) the account earns interest compounded daily?

N = 10×365
P/Y = 365
$2717.91 (incr of $2.24)

Effective interest rate r_{eff} can be found with a FINANCE command

Eff(I%, m)

Example

You find a bank that pays 8.15% annual interest compounded quarterly and another that pays 8.1% annual interest compounded daily. Which is a better deal?

eff(8.15, 4) = 8.4025

eff(8.1, 365) = 8.4361 * better for a savings acct

If interest is compounded continuously, then the future value of an investment of P dollars at an annual interest of r (expressed as a decimal) for t years is $F = Pe^{rt}$.

Example

You deposit \$1000 into an account that pays 10% annual interest. How much is in the account after 10 years if the interest is compounded continuously?

$F = 1000e^{(.1)(10)} = \$2718.28$ (incr of \$.37 over daily)

Example

You wish to have $60,000 available for your child's college expenses. You find an account that pays 7% annual interest compounded monthly.

How much needs to be deposited now to reach this goal if you want to have the money available in 18 years?

N=18x12=216
I= 7
PV= ? → $17081.66
PMT= 0
FV = 60000
P/Y= 12

How much interest was earned?

60000 − 17081.66 = $42918.34

What if you wanted the money available in 8 years?

N= 8x12= 96

→ $34,328.35

How much interest was earned?

60000 − 34328.35 = $25671.65

Example

You deposit $4000 in an account that earns 5% annual interest compounded weekly. How long does it take for the account to double in value?

N= ?
I= 5
PV= 4000
PMT= 0
FV= -8000
P/Y= 52

a sign change

N= 721.22 weeks
just under 14 years

Example

You deposited $200 in a savings account 10 years ago. It is now worth $350. You know the account paid annual interest that was compounded annually, but you don't remember the interest rate promised. What was the annual interest rate for the account?

N= 10
I= ?
$PV= 200
$PMT= 0
$FV= -350
P/Y= 1

5.76%

Example

Your retirement account was worth $16,000 on January 1 and worth $15,000 two years later. Find the annual interest rate.

N= 2
I= ?
PV= 16000
PMT= 0
FV= -15000
P/Y= 1

I= -3.18%

Example

To save for retirement, you deposit $2000 every year in an account that earns 6% annual interest compounded annually.

(a) How much will you have after 10 years? How much interest did you earn?

N = 10×1 = 10 PMT = 2000
I = 6 FV = ?
PV = 0 P/Y = 1

→ FV = 26,361.59

deposited 10 × 2000 = $20,000

Int = 26361.59 − 20000 = $6,361.59

(b) How much will you have after 25 years? How much interest did you earn?

N = 25

→ $109,729.02 − 25(2000) = $59,729.02 in interest

(c) How much will you have after 40 years? How much interest did you earn?

N = 40

→ $309,523.93 − 40(2000) = $229,523.93

An ***annuity*** is an account to which regular payments are made. An annuity that is ***certain and simple*** has the following properties:

1. The payments are made at fixed time intervals
2. The periodic payments are of equal size
3. The payments are made at the end of the interval
4. The interest is paid at the end of the interval

Many loans and savings plans are certain and simple annuities

Any account that is established to meet a future need is called a ***sinking fund***.

Example

You wish to have $60,000 available for your child's college expenses. You find an account that pays 7% annual interest compounded monthly. How much needs to be deposited every month to reach this goal if you want to have the money available in 18 years? Assume that the monthly payments are of equal size. How much interest does the account earn?

N= 12x18= 216
I= 7
PV= 0
PMT= ? → $139.30
FV= 60000
P/Y= 12

Int = 60000 − 216(139.30)
= $29,911.20 in interest

Example

You start a savings account with the goal of saving $5000 in 4 years by making regular quarterly payments to an account that earns 8% annual interest compounded quarterly.

(a) How large are the quarterly payments?

N= 4x4=16
I= 8
PV= 0
PMT= ?
FV=5000
P/Y= 4

PMT= $268.25

(b) How much interest is earned in all?

5000 − 16 x 268.25= $708

Example

A cello costs \$574 to buy or \$40.58 per month on a 2 year rent-to-own plan. What is the interest rate charged?

N= 24 \$PMT= 40.58 → I = 56.96%
I= ? \$FV = 0
\$PV= -574 P/Y=12

Example

You owe \$6000 on a credit card that charges 21% annual interest compounded monthly on the unpaid balance.

(a) If you make monthly payments of \$120, how long will it take to pay it off?

N= ? PMT= 120 → N = 119.86 mo
I= 21 FV= 0 ⇒ 10 years
what you owe * PV= -6000 P/Y= 12

(b) How much interest is paid in all?

120 x 120 = 14400
− 6000
\$8400

60 PMT made
60 PMT to go

(c) How much do you owe after 5 years of making payments?

N= 60 (PMTS left) PMT= 120 → PV= \$4435.68 still owe
I=21 FV=0
PV= ? P/Y= 12

Example

You want a pair of diamond earrings that cost \$800. You decide to stop buying daily lattes and instead save the \$5 every day in an account that pays 6% annual interest compounded daily. How long until you have the money for the earring?

Example

NOTE: $\frac{800}{5}$ = 160 days if it earns no interest

N= ? 158 days PMT = 5 NOT −162
I= 6 PV = -800
PV= 0 P/Y= 365

$20 \times 30000 = 600{,}000$

When you retire you would like to have $30,000 per year. Assuming that you will need to have 20 payments starting right now, how much do you need to deposit into an account that pays 5% annual interest that is compounded annually?

N= 20
I= 5
PV= ? → $373,866.31
PMT= 30000
FV= 0
P/Y= 1

Example

You wish to purchase a house that costs $130,000. You make a down payment of $20,000 and finance the remainder for 30 years at 5.1% annual interest compounded monthly on the unpaid balance.

(a) How large are the monthly payments?

N= 12x30=360
I= 5.1
PV= 110000
PMT= ? → $597.24
FV= 0
P/Y= 12

afford $600/mo mtgpmt
N=360
I=5.1
PV= ?
PMT= 600
FV= 0
P/Y= 12

(b) How much interest is paid in all?

360 (597.24) = 215,006.40
−110,000
$105,006.40 in interest

(c) How much of the first payment is interest?

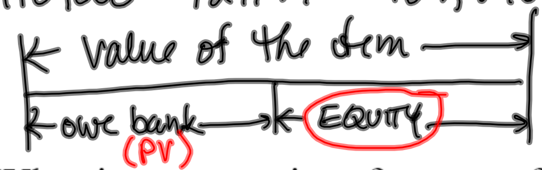
Monthly interest

$110{,}000 * \frac{0.051}{12} = \$467.50$ owed in interest

$597.24 - 467.50 = \$129.74$

(d) How much do you still owe the bank after your first payment?

$110{,}000 - 129.74 = 109{,}870.26$



(e) What is your equity after your first payment?

$130{,}000 - 109{,}870.26 = 20{,}129.74$

(f) What is your equity after your second payment?

Interest owed $109{,}870.26 * 0.051/12 = \$466.95$

$597.24 - 466.95 = \$130.29$ pay down the loan

$109{,}870.26 - 130.29 = 109{,}739.97$ still owed

$EQ = 130{,}000 - 109{,}739.97 = 20{,}260.03$

(g) What is your equity after your 12th payment?

12 pmts made, $360 - 12 = 348$ pmts to go

N = 348
I = 5.1
PV = ?
PMT = 597.24
FV = 0
P/Y = 12

$\rightarrow$ 108,406.15 still owe

$EQ = 130\,000 - 108406.15$

$= \$21{,}593.85$

Example N= 8, I= 12, PV= 3000, PMT= ?, FV= 0, P/Y= 4 ⇒ PMT= 427.37

You borrow $3000 at 12% annual interest compounded quarterly for 2 years. Show in a table how much of each payment is interest, how much is paid towards the principal, how much of the loan is paid off and how much is still owed each quarter.

$.12/4 = 0.03$

Paid off

end of period	# PMT left	PMT ($)	interest ($)	towards principal	outstanding principal	"equity"
0	8		$3000 \times \frac{.12}{4}$		3000	0
1	7	427.37	90	337.37	2662.63	337.37
2		"	79.88	347.49	2315.14	684.86
3		"				
4						
5						
6						
7						
8						

Example

For the house purchased earlier, fill in the following lines of an amortization table.

end of period	# PMT left	PMT ($)	interest ($)	towards principal	outstanding principal	equity
0						
1						
2						
12						
60						
120						
180						
240						
300						