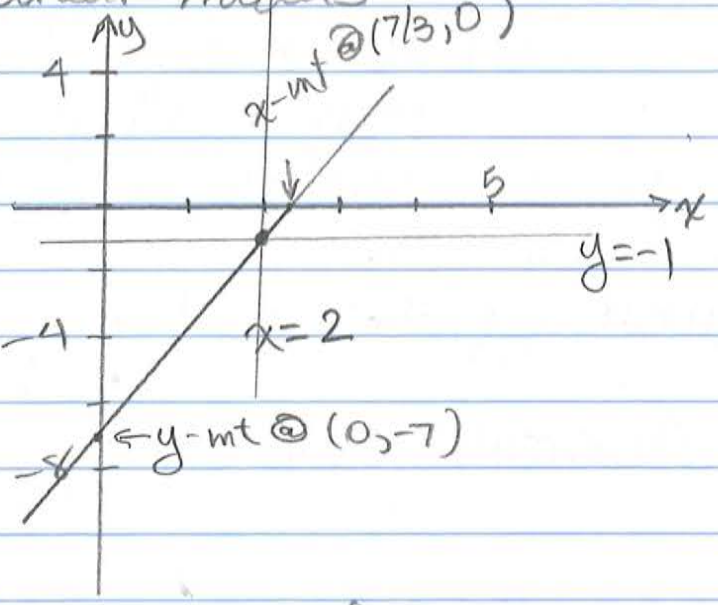
I.1
I.2

Part I – Lines and Linear models

1. $y - y_1 = m(x - x_1)$
$y - (-1) = 3(x - 2)$
$y + 1 = 3x - 6$
$y = 3x - 7$
$x = 0 \rightarrow y = -7$
$y = 0 \rightarrow x = 7/3$



② Demand is (x, p) : $(20, 45)$ and $(20+30, 45-15)$
$= (50, 30)$

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta p}{\Delta x} = \frac{45-30}{20-50} = \frac{15}{-30} = -\frac{1}{2}$$

$y - y_1 = m(x - x_1)$ or $p - p_1 = m(x - x_1)$

$p - 45 = (-\frac{1}{2})(x - 20) \Rightarrow p = D(x) = -\frac{1}{2}x + 55$
p in \$, x = # purses

50 purses $\rightarrow x = 50 \rightarrow p = (-\frac{1}{2})(50) + 55 = 30$
price is \$30 per purse

I.3
I.4

3. $C(x) = cx + F$ with c the cost to make 1 item and F the fixed costs

$C(x) = 0.5x + 600$ with x = # of stickers and C the total cost in $ to make them

$R = 2x$ = total revenue in $ from selling x stickers

BE @ $R = C$ or $P = 0$

$\Rightarrow 2x = 0.5x + 600 \Rightarrow x = 400$ } (400, 800)

$R(400) = 800 = C(400)$ }

At the break even point 400 bumper stickers will be made and sold. The total cost to produce these stickers is $800 which is also the total revenue from selling the stickers.

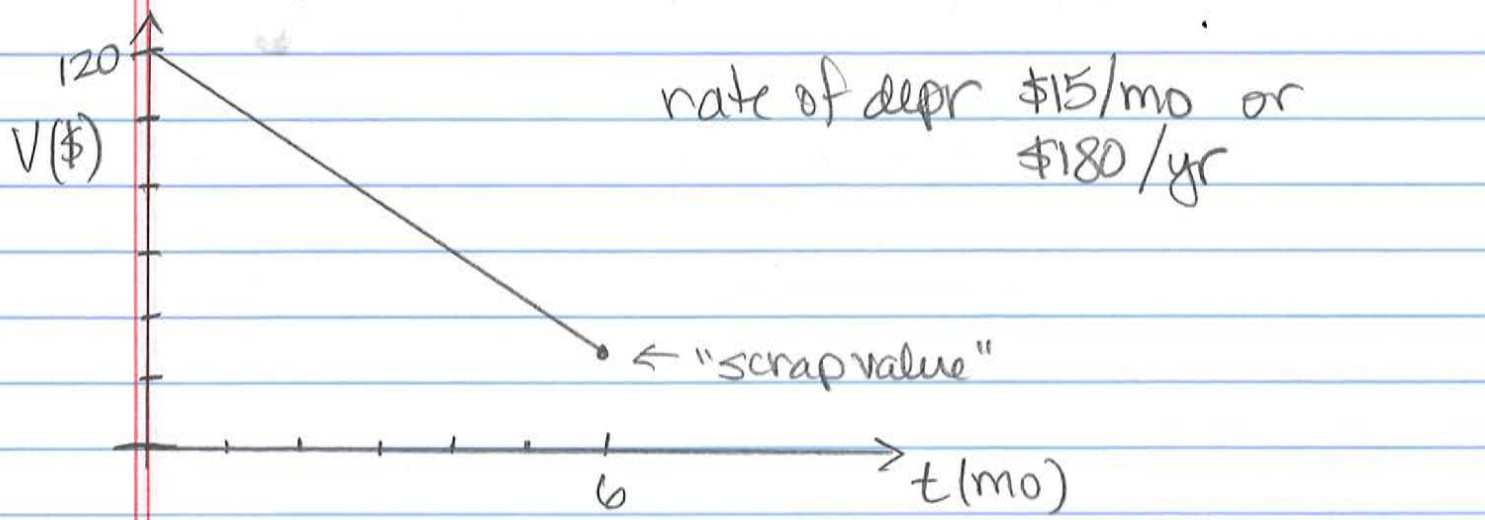
4. (t, V): (0, 120) and (6, 30) with t in months and V in dollars

$\Rightarrow V = -15t + 120$

OR

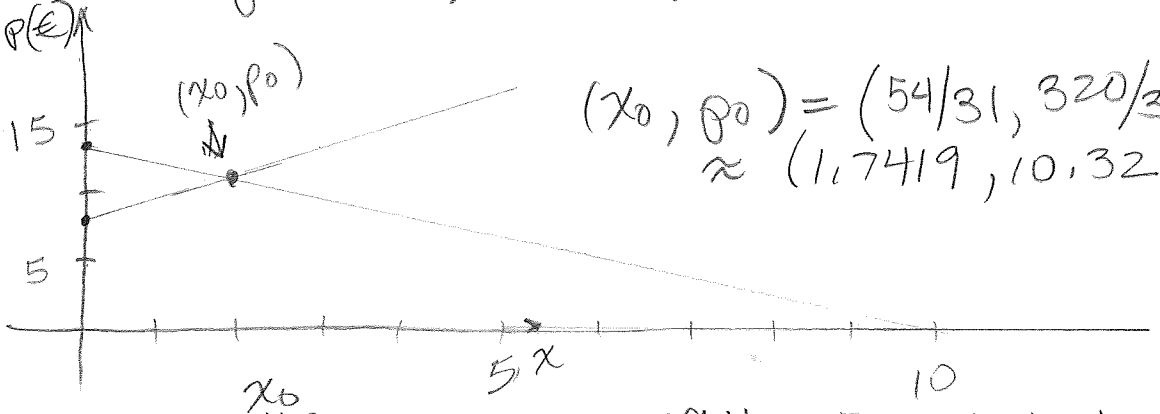
(t, V): (0, 120) and (.5, 30) with t in years

$\Rightarrow V = -180t + 120$



I.5
I.6

5 $3p - 4x = 24$ (S) x in th of lamps
$4p + 5x = 50$ (D) p price per lamp in €
supply $= p = S(x) = (4/3)x + 8$ } window 0 to 15
demand $= p = D(x) = (-5/4)x + 12.5$ } 0 to 20



$(x_0, p_0) = (54/31, 320/31)$
$\approx (1.7419, 10.3226)$

qty $= 1.7419\ldots \times 1000 = 1741.935 \Rightarrow 1742$ lamps
price $= 10.32$ € or 10.32 euros
At equilibrium point the supply of lamps is equal to the demand for lamps. So 1742 lamps are produced and sold at a price of 10.32 euros each.

6. $3y - 4x = 16 \Rightarrow 3y = 4x + 16 \Rightarrow y = (4/3)x + 16/3$

a) parallel $m = 4/3$, $y - 4 = (4/3)(x - (-3))$
$y = (4/3)x + 4 \quad +4 = (4/3)x + 8$

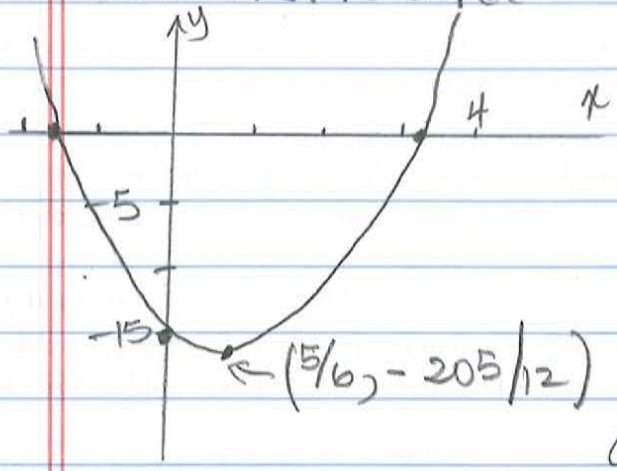
b) perp $m = -1/(4/3) = -3/4$
$y - (-6) = -3/4\,(x - 4)$
$y + 6 = -3/4\,x + 3$
$y = -3/4\,x - 3$

I.7
I.8

7. $x=0 \Rightarrow y=-15$

$y=0 \Rightarrow x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(3)(-15)}}{2(3)}=\dfrac{5\pm\sqrt{205}}{6}$

$x\approx 3.2196$ and $x\approx -1.5530$



vertex $x=-\dfrac{(-5)}{2(3)}=\dfrac{5}{6}$

$y=3(5/6)^2-5(5/6)-15$

$=-205/12$

≈ -17.0833

a minimum

8. $P=R-C$. $R=px=(-2.2x+30)x=-2.2x^2+30x$

$P=-2.2x^2+30x-1.5x-7=-2.2x^2+28.5x-7$

profit in \$, x is th. of clipboards

vertex @ $x=-28.5/2(-2.2)=-285/44\approx 6.4772$

$*1000 \Rightarrow 6477$ clipboards

max profit is $-2.2(6.477)^2+28.5(6.477)-7=85.301$

$\Rightarrow \$85.30$ in profit .

Part 2 - Systems of Linear Equations

① no soln ⇒ parallel lines ⇒ same slope & diff intercept

$3x - y = 4$ or $y = 3x - 4 \Rightarrow m = 3$

$-6x + ky = 10$ or $y = (6/k)x + 10/k \Rightarrow m = -6/k$

$6/k = 3 \Rightarrow k = 2$. check

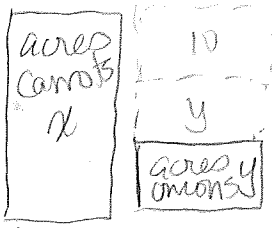
$10/k = 10/2 = 5 \neq -4$ so diff y-int

② x = # of acres of carrots

y = # of acres of onions

$x + y = 100$ total acres

$x = 2y + 10$ ratio carrots/onions



Solve graphically, algebraically or with RREF

$\Rightarrow x = 70, y = 30$

Plant 30 acres of onions

③ x = # of shares in ABC

y = # of shares in XYZ

$32x + 23y = 10{,}100$ total money

$1.2x + 1.4y = 540$ total div

$$\left[\begin{array}{cc|c} 32 & 23 & 10{,}100 \\ 1.2 & 1.4 & 540 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 100 \\ 0 & 1 & 300 \end{array}\right] \quad x = 100,\ y = 300$$

Invest 100 sh of ABC and 300 sh of XYZ

x = # of $ in ABC

y = # of $ in ABC

$x + y = 10{,}100$

$\left(\frac{x}{32}\right)(1.2) + \left(\frac{y}{23}\right)(1.4) = 540$

$$\left[\begin{array}{cc|c} 1 & 1 & 10{,}100 \\ 1.2/32 & 1.4/23 & 540 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 3200 \\ 0 & 1 & 6900 \end{array}\right] \quad x = 3200,\ y = 6900$$

Invest $3200 in ABC and $6900 in XYZ

(4)

$$\begin{array}{c} x \quad y \\ \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & b \end{array}\right] \end{array}$$

any value of b

however if $b \neq 0$ then no solution
if $b = 0$, solution is $(t, 0)$ t any #

(5)

$$\begin{array}{l} 2x + 3y + z = 1 \\ x + y + z = 3 \\ 3x + 4y + 2z = 4 \end{array} \longleftrightarrow \begin{array}{c} x \quad y \quad z \quad = \quad \# \\ \left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 3 & 4 & 2 & 4 \end{array}\right] \end{array} \xrightarrow{\text{RREF}}$$

$$\begin{array}{c} x \quad y \quad z \overset{t}{=} \# \\ \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} 1x + 0y + 2z = 8 \text{ or } x + 2t = 8 \\ 0x + 1y - 1z = -5 \text{ or } y - t = -5 \\ 0x + 0y + 0z = 0 \end{array}$$

$(x, y, z) = (8 - 2t, t - 5, t)$ t any #

6. $\begin{matrix} -\frac{1}{2}x + 3y = 15 \\ 2x + 5y = 8 \end{matrix} \rightarrow \left[\begin{array}{cc|c} -\frac{1}{2} & 3 & 15 \\ 2 & 5 & 8 \end{array}\right]$ $\xrightarrow{-2R_1 \rightarrow R_1}$

$\left[\begin{array}{cc|c} 1 & -6 & -30 \\ 2 & 5 & 8 \end{array}\right]$ $\xrightarrow{-2R_1 + R_2 \rightarrow R_2}$ $\left[\begin{array}{cc|c} 1 & -6 & -30 \\ 0 & 17 & 68 \end{array}\right]$

$\begin{matrix} +(1)(-2) & +(-6)(-2) & +(-30)(-2) \\ \hline 0 & 17 & 68 \end{matrix}$

$\xrightarrow{\frac{1}{17}R_2 \rightarrow R_2}$ $\left[\begin{array}{cc|c} 1 & -6 & -30 \\ 0 & 1 & 4 \end{array}\right]$ $\xrightarrow{6R_2 + R_1 \rightarrow R_1}$

$\left[\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & 4 \end{array}\right]$ $\quad x = -6$, $y = 4$

7. x = # of goldfish
y = # of bluefish
z = # of greenfish

# gold f.	# green f.
x	z

$2.5x + 2y + 1z = 20$ total # to buy fish

$x = 2z$ ratio gold to green fish

$\left[\begin{array}{ccc|c} 2.5 & 2 & 1 & 20 \\ 1 & 0 & -2 & 0 \end{array}\right]$ $\xrightarrow{\text{ref}}$ $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 10 \end{array}\right]$ $\begin{matrix} x - 2t = 0 \\ y + 3t = 10 \end{matrix}$

$(x, y, z) = (2t, 10 - 3t, t)$ t = # of green fish

$t = 0$ $(0, 10, 0)$ buy 0 gold, 10 blue, and 0 greenfish
$t = 1$ $(2, 7, 1)$ 2 7 1
$t = 2$ $(4, 4, 2)$ 4 4 2
$t = 3$ $(6, 1, 3)$ 6 1 3

Or buy 0, 1, 2 or 3 green fish with the number of gold being twice the number of green and ...

(8) (pt)

$$\left[\begin{array}{ccc|c} 1.2 & 0.8 & 1 & 12 \\ -3 & -2 & -2.5 & k \end{array}\right] \xrightarrow{1/1.2\,R_1 \to R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 5/6 & 10 \\ -3 & -2 & -2.5 & k \end{array}\right] \xrightarrow{3R_1 + R_2 \to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 5/6 & 10 \\ 0 & 0 & 0 & k+30 \end{array}\right]$$

$x + 2/3\,y + 5/6\,z = 10$

$0 = k + 30$

$\Rightarrow k = -30$ other values $\Rightarrow$ no soln

Part 3- Matrices

(1) $\begin{bmatrix} 5\cdot 2 & 5\cdot 4 \\ 5\cdot -1 & 5\cdot a \end{bmatrix} + \begin{bmatrix} -4 & c \\ b & 3 \end{bmatrix} = \begin{bmatrix} d & 0 \\ 1 & 6 \end{bmatrix}$

$\begin{bmatrix} 10-4 & 20+c \\ -5+b & 5a+3 \end{bmatrix} = \begin{bmatrix} d & 0 \\ 1 & 6 \end{bmatrix}$

$6 = d$

$20+c = 0 \Rightarrow c = -20$

$-5+b = 1 \Rightarrow b = 6$

$5a+3 = 6 \Rightarrow 5a = 3 \Rightarrow a = 3/5 = .6$

$= B\begin{pmatrix} B & L & D \\ 3 & 6 & 10 \end{pmatrix}$ $= \begin{matrix} \$ \\ B \\ L \\ D \end{matrix}\begin{pmatrix} 3 \\ 6 \\ 10 \end{pmatrix}$

(2) A is 3x3 B_1 is 1x3 and B_2 is 3x1

R wee have 3 pieces of unformation (ave $ for each rest))

$A \cdot B_1$ is $(3\times3)\cdot(1\times3)$ DNE

$B_1 \cdot A$ is $(1\times3)\cdot(3\times3) = (1\times3)$ maybe

$A \cdot B_2$ is $(3\times3)\cdot(3\times1) = (3\times1)$ maybe

$B_2 A$ is $(3\times1)\cdot(3\times3)$ DNE

$$A \cdot B_2 = \begin{matrix} \\ B \\ L \\ D \end{matrix}\begin{pmatrix} \text{I} & \text{II} & \text{III} \\ 66 & 300 & 250 \\ & & \\ & & \end{pmatrix} \begin{matrix} \$ \\ B \\ L \\ D \end{matrix}\begin{pmatrix} 3 \\ 6 \\ 10 \end{pmatrix}$$ meaningless

$$B_1 A = \$\begin{pmatrix} B & L & D \\ 3 & 6 & 10 \end{pmatrix} \begin{matrix} \\ B \\ L \\ D \end{matrix}\begin{pmatrix} \text{I} & \text{II} & \text{III} \\ 66 & & \\ 150 & & \\ 50 & & \end{pmatrix}$$

$$= \$\begin{pmatrix} \text{I} & \text{II} & \text{III} \\ 1598 & 8100 & 5350 \end{pmatrix}$$

③ $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1x+3y \\ 2x-y \end{bmatrix} \Rightarrow \begin{matrix} x+3y=4 \\ 2x-y=0 \end{matrix}$

④ YX is $(3\times1)\cdot(4\times3)$ and DNE

$$XY = \begin{matrix} \\ \text{meat} \\ \text{fruit} \\ \text{grain} \\ \text{dairy} \end{matrix}\begin{matrix} \begin{matrix} \text{fat} & \text{carb} & \text{pro} \end{matrix} \\ \begin{pmatrix} 5 & 0 & 7 \\ 0 & 10 & 1 \\ 0 & 15 & 2 \\ 10 & 12 & 8 \end{pmatrix} \end{matrix} \cdot \begin{matrix} \\ \text{fat} \\ \text{carb} \\ \text{pro} \end{matrix}\begin{matrix} \begin{matrix} \text{cal} \end{matrix} \\ \begin{pmatrix} 8 \\ 4 \\ 5 \end{pmatrix} \end{matrix}$$

$$= \begin{matrix} \\ \text{meat} \\ \text{fruit} \\ \text{grain} \\ \text{dairy} \end{matrix}\begin{matrix} \begin{matrix} \text{cal} \end{matrix} \\ \begin{pmatrix} 5\cdot8+0\cdot4+7\cdot5 \\ 0\cdot8+10\cdot4+1\cdot5 \\ 0\cdot8+15\cdot4+2\cdot5 \\ 10\cdot8+12\cdot4+8\cdot5 \end{pmatrix} \end{matrix} \begin{matrix} \\ =75 \\ =45 \\ =70 \\ =168 \end{matrix}$$

= total cal from fat, carb and protein in one "unit" of each food

⑤ a) A+D is $(4\times4)+(2\times4)$ DNE dim mismatch
b) A+B is $(4\times4)+(4\times4)$ is a (4×4)
c) $C+D^T$ is $(4\times2)+(4\times2)$ is a (4×2)
d) BI is $(4\times4)\cdot(4\times4)$ is a 4×4
e) BC is $(2\times2)\cdot(4\times2)$ DNE (dim mismatch)
f) BD is $(2\times2)(2\times4)=(2\times4)$
g) AC is $(4\times4)(4\times2)$ is a 4×2
h) D^2 is $(2\times4)\cdot(2\times4)$ DNE dim mismatch
i) B^2 is $(4\times4)\cdot(4\times4)$ is a 4×4
j) CD is $(4\times2)\cdot(2\times4)$ is a 4×4