CHAPTER 16 – IDENTIFICATION NUMBERS

Consider the UPC code on a can of RO★TEL tomatoes

The scanner is not working so the clerk enters the numbers by hand as

0 64144 28263 2

and this is invalid even though the product code for the mild version of this is 28263. What happened?

The UPC codes use a ***check digit*** to minimize scanning errors. A check digit is a digit included in a code to help detect errors.

For a UPC code $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}$ has a_{12} chosen so that the sum

$$3(a_1+a_3+a_5+a_7+a_9+a_{11})+1(a_2+a_4+a_6+a_8+a_{10})$$

is evenly divisibly by 10. What is the check digit for the mild RO★TEL?

1 2 3 4 5 6 7 8 9 10 11
0 6 4 1 4 4 2 8 2 6 3

$3(0+4+4+2+2+3)+1(6+1+4+8+6)=70 \Rightarrow$ $70+a_{12}$ divisible by 10

$\Rightarrow a_{12}=0$

The numbers 20, 60, and 100 are all evenly divisible by 10 so can we find a way to talk about numbers when we only care about the remainders?

Definition: Congruence Modulo m

Let a, b, and m be integers with $m \geq 2$. Then a is congruent to b modulo m, written

$$a \equiv b \bmod m$$

means that m evenly divides $a - b$.

Determine if the congruences below are true or false:

$25 \equiv 1 \bmod 6$ TRUE

$\frac{25-1}{6} = 4 \text{ R } 0$

$100 \equiv 20 \bmod 10$ TRUE

$\frac{100-20}{10} = 8 \text{ R } 0$

$52 \equiv 0 \bmod 13$ TRUE

$\frac{52-0}{13} = 4 \text{ R } 0$

$75 \equiv 7 \bmod 5$ false

$\frac{75-7}{5} = 13 \text{ R } 3$

x* mod *y is equal to the remainder when you divide x by y.

Find the following values:

(a) 34 mod 5 = 4 $\quad \frac{34}{5} = 6 \text{ R } 4$

(b) 78 mod 11 = 1 $\quad \frac{78}{11} = 7 \text{ R } 1$

(c) 13 mod 15 = 13 $\quad \frac{13}{15} = 0 \text{ R } 13$

(d) 12 mod 2 = 0 $\quad \frac{12}{2} = 6 \text{ R } 0$

Types of errors when dealing with identification numbers:

- Replacing one digit with a different digit (single digit error)
  - *ac* entered rather than *ab*
- Transposing two adjacent digits (adjacent transposition error)
  - *ba* entered rather than *ab*
- Transposing a sequence of digits (jump transposition error)
  - *cba* entered rather than *abc*

Note that some of the digits in the UPC code are multiplied by 3. Those digits had a ***weight*** of 3.

A code $a_1a_2a_3a_4a_5a_6$ uses the last digit as a check digit. The check digit is found using the formula

$$a_6 = a_1 + a_3 + 3(a_2 + a_4) \bmod 10$$

+5a5 ⟵ (*)

(a) What is the check digit for the code 23714?

$a_6 = 2 + 7 + 3(3+1) + 4(5) = 41 \bmod 10$

$a_6 = 1$

(b) Find the value of the missing digit x in the code $46x782$

0,1,2,3,4,5,6,7,8,9 — what has R2 after divide by 10? — a_6

$a_6 = 4 + x + 3(6+7) + 5(8) = 83 + x = 92$ ⇒ $x = 9$

$= 2, 12, 22, 32, 42, 52, 62, 72, 82, 92, 102, \ldots$

x NOT NEG

(c) Will this code find an error if the first digit is entered incorrectly?

correct code and incorrect code have the same a_6

(correct) $a_1a_2a_3a_4a_5$ VS (wrong) $b_1a_2a_3a_4a_5$ ⇒ do these have the same ck digit?

$a_6 = a_1 + a_3 + 3(a_2 + a_4) + 5a_5 \bmod 10$

$b_6 = b_1 + a_3 + 3(a_2 + a_4) + 5a_5 \bmod 10$

} check if $a_6 - b_6 = 0 \bmod 10$

$a_6 - b_6 = a_1 + a_3 + 3(a_2+a_4) + 5a_5 - b_1 - a_3 - 3(a_2+a_4) - 5a_5 = a_1 - b_1 \bmod 10$

$= 0$?

0 to 9, 0 to 9

$9-0=9$, $8-4=4$, etc: $|a_1 - b_1| = 0, 1, 2, \ldots, 9$

$= 0$ if $a_1 = b_1$

if $a_1 = 2$ and $b_1 = 7$

$= 0$ only if $a_1 = b_1$ (no error) so catches all errors in the 1st position

A code is given by $a_1a_2a_3$ where a_3 is the check digit and

$$a_3 = a_1 + 4\,a_2 \mod 9$$

(a) Will this check digit find all transposition errors?

$\Rightarrow a_1a_2$ entered as a_2a_1 so find if $a_3 = b_3$ (check digits are the same)

$a_3 = a_1 + 4a_2 \mod 9$ and $b_3 = a_2 + 4a_1$

$a_3 - b_3 = a_1 + 4a_2 - a_2 - 4a_1 = 3a_2 - 3a_1 = 3(a_2 - a_1) = 0 \mod 9$ 0, 9, 18, 27, 36, etc

no error 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$a_2 - a_1 = 3 \Rightarrow a_1 = 2$ and $a_2 = 5$ or loads more

(b) Will this check digit find all single digit errors in the first position?

a_1a_2 entered as b_1a_2

$a_3 = a_1 + 4a_2 \mod 9$ and $b_3 = b_1 + 4a_2$ 0, 9, 18, 27, ...

$a_3 - b_3 = a_1 + 4a_2 - b_1 - 4a_2 = a_1 - b_1 \mod 9 = 0 \mod 9$

1, 2, 3, 4, 5, 6, 7, 8, 9 ← won't catch errors when digits differ by 9 ($a_1 = 0$, $b_1 = 9$ or $a_1 = 9$, $b_1 = 0$)

(c) Will this check digit find all single digit errors in the second position?

a_1a_2 entered as a_1b_2 : $a_3 = a_1 + 4a_2$ and $b_3 = a_1 + 4b_2$

$a_3 - b_3 = a_1 + 4a_2 - a_1 - 4b_2 = 4(a_2 - b_2) = 0 \mod 9$ } 0, 9, 18, 27, 36, 45, etc

no error 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$a_3 = a_1 + 4a_2 \mod 11$? check digit for 22?

$a_3 = 2 + 4(2) \mod 11 = 10 \mod 11 \Rightarrow$ 10

2 DIGITS! $\Rightarrow$ use X

Data can be encoded in identification numbers.

Jan 1 : 001 , Jan 31: 031

The last 5 digits of Illinois driver's license numbers are based on the driver's birthday and gender. For a man, the last 5 digits are the birth year followed by the day of the year based on each month having 31 days. For a woman, 600 is added to the number.

$31(m-1)+d$

(a) What would the last 5 digits of an of Illinois driver's license number look like for a man born on February 12, 1967? 6 7 0 4 3

$31(2-1)+12=43$

(b) What do you know about a person who has the last 5 digits 10642?

Born 1910 or 2010? woman. birth day is 042

Jan 1 : 001

Feb 1 : 032

Mar 1 : 063

Feb 11

(c) What do you know about a person who has the last 5 digits 90373?

Man born 1890 or 1990

Dec 31 : $31(12-1)+31=372$

FAKE

SAMPLE EXAM QUESTIONS FROM CHAPTER 16

1. Determine the check digit that should be appended to the identification number 634498, if the check digit is the number needed to bring the total of all the digits to a multiple of 10.

(A) The code is invalid (B) 6 (C) 8 (D) 4

(E) None of these

$6+3+4+4+9+8 = 34$

2. Which, if any, of the statements below are true? Mark all correct answers.

T (A) $101 \equiv 1 \bmod 2$: $\frac{101-1}{2} = 50$ R0

T (B) $77 \equiv 0 \bmod 11$ $\frac{77-0}{11} = 7$ R0

T (C) $49 \equiv 1 \bmod 12$ $\frac{49-1}{12} = \frac{48}{12} = 4$ R0

F (D) $39 \equiv 5 \bmod 5$ $\frac{39-5}{5} = 6$ R 4

(E) None of these are true.

3. The number 4320 is accidentally entered as 4321.
What type of error is this?

(A) A transposition error

(B) A jump transposition error

(C) A single digit error

(D) A baseball error

(E) None of these

4. The last three digits of a man's ID number are the birth day of the year based on each month of the year having 35 days. If the person is a woman, 500 is added to the birth day.

$35(m-1)+d$ Feb 1: 036

↑ so Jan 1 is 001

(a) What are the last three digits of a man's ID number if he was born on October 8th?

$35(10-1)+8 = 323$

(b) What do you know about a person if the last three digits of the person's ID number are 503?

Woman $\Rightarrow$ 003 $\Rightarrow$ Jan 3

(c) What do you know about a person if the last three digits of the person's ID number is 420?

Man? but Dec 31 is $35(12-1)+31 = 416$

NOT VALID

5. A code is given by $a_1a_2a_3a_4$ where a_4 is the check digit. The check digit is $a_4 = 7a_1 + 2a_2 + 5a_3 \bmod 9$.

(a) Determine the value of x in the code $2x45$, given that the check digit is valid.

1 2 3 4 correct

$a_4 = 7(2) + 2(x) + 5(4) = 5 \bmod 9 \Rightarrow 5, 14, \ldots 32, 41, 50, \text{etc}$

$14 + 2x + 20 = 34 + 2x$

$x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

(b) Determine if the check digit will find all single digit errors in the third position.

$a_1a_2a_3$ entered as $a_1a_2b_3$; $a_4 = 7a_1 + 2a_2 + 5a_3$ and $b_4 = 7a_1 + 2a_2 + 5b_3$

$a_4 - b_4 = 7a_1 + 2a_2 + 5a_3 - 7a_1 - 2a_2 - 5b_3 = 5(a_3 - b_3) = 0 \bmod 9?$

$0, 9, 18, 27, 36, 45, \ldots$

$1, 2, 3, 4, 5, 6, 7, 8, 9$

won't catch errors when $|a_3 - b_3| = 9$

(c) Determine if the check digit will find all transposition errors in the second and third positions.

$a_4 = 7a_1 + 2a_2 + 5a_3$, $b_4 = 7a_1 + 2a_3 + 5a_2$

$a_1a_2a_3$ entered as $a_1a_3a_2$

$a_4 - b_4 = 7a_1 + 2a_2 + 5a_3 - 7a_1 - 2a_3 - 5a_2 = 3a_3 - 3a_2 = 3(a_3 - a_2) = 0 \bmod 9$

$1, 2, 3, 4, 5, 6, 7, 8, 9$

$0, 9, 18, 27, 36, 45, \ldots$

This will not catch errors when $|a_2 - a_3| = 3$ or 6 or 9