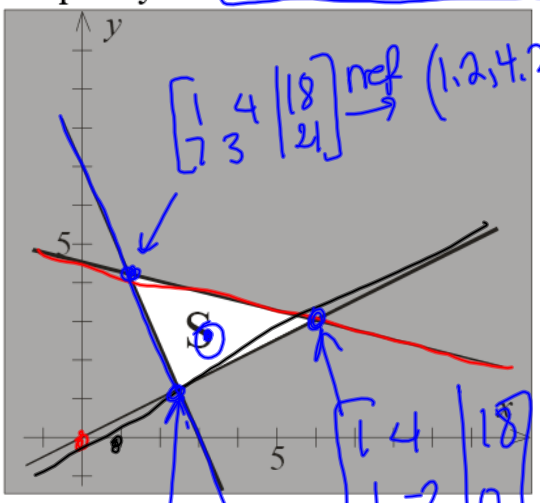
Exam 2 Review Questions

LINEAR PROGRAMMING

1. The graph below shows the feasible region in white. Circle the correct inequality and find the exact values of all corner points.



$x + 4y \leq 18$ $\geq$

$7x + 3y \leq 21$ $\geq$

$x - 2y \leq 0$ $\geq$

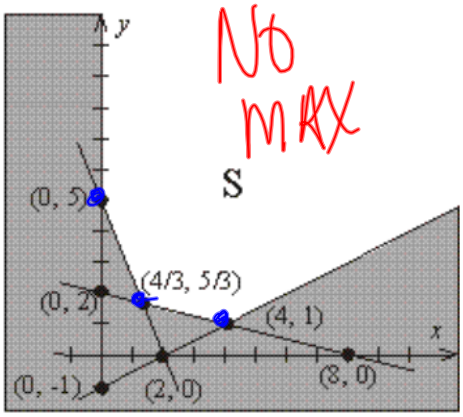
$\begin{bmatrix} 1 & 4 & 18 \\ 7 & 3 & 21 \end{bmatrix}$ ref $\rightarrow$ (1.2, 4.2)

$\begin{bmatrix} 1 & 4 & 18 \\ 1 & -2 & 0 \end{bmatrix}$ ref $\rightarrow$ (6, 3)

$\begin{bmatrix} 1 & -2 & 0 \\ 7 & 3 & 21 \end{bmatrix}$ ref $\rightarrow$ $\left(\frac{42}{17}, \frac{21}{17}\right)$

(3,3) is true for everyone

2. What are the maximum and minimum values of $f(x, y) = 3x + 4y$, if any, on the region shown to the right?



No MAX

vertex	$f = 3x + 4y$
(0,5)	20
(4/3, 5/3)	10 2/3 $\rightarrow$ min
(4,1)	16

3. A diet planner is trying to determine how much chicken and potatoes can be used in a packaged dinner. The nutritional requirements produce a feasible region that is unbounded with corners at (0, 40), (5, 20), (10, 10) and (30, 0). If x is the number of ounces of chicken and y is the number of ounces of potatoes, then the calories in the meal is given by

$$C = 42x + 21y.$$

Determine how much chicken and potato should be used in a meal to meet the nutritional requirements with a minimum number of calories.

Vertex	$C = 42x + 21y$
(0, 40)	840
(5, 20)	630 } min
(10, 10)	630
(30, 0)	1260

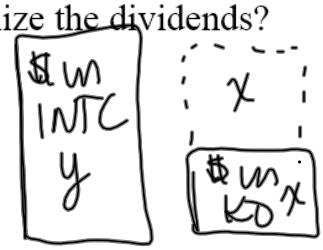
Run on $630 = 42x + 21y$

for $5 \le x \le 10$

$y = 30 - 2x$

Use $5 \le x \le 10$ oz of chicken and $y = 30 - 2x$ oz of potatoes for a total of 630 calories

4. Set up the following Linear Programming Problem. *DO NOT SOLVE*. Mazie has at most \$12000 to invest in three different stocks. The KO company costs \$42.00 per share and pays dividends of \$1.25 per share. The INTC company costs \$21.00 per share and pays dividends of \$0.40 per share. The MCD company costs \$35.00 per share and pays \$0.67 per share in dividends. Mazie has given her broker the following instructions: Invest at least twice as much money in INTC as in KO. Also, no more than 25% of the total invested should be in MCD. How should Mazie invest her money to maximize the dividends?



x = \$ in KO
y = \$ in INTC
z = \$ in MCD
D = div in \$

$$\text{Max } D = 1.25\left(\frac{x}{42}\right) + .4\left(\frac{y}{21}\right) + .67\left(\frac{z}{35}\right)$$

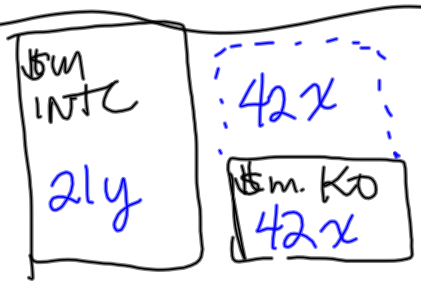
SUB. TO

$x + y + z \le 12000$ (total \$)

$y \ge 2x$ (ratio INTC & KO)

$z \le .25(x + y + z)$ limit on MCD

$x, y, z \ge 0$



x = # of sh of KO
y = # of sh of INTC
z = # of sh of MCD
D = div in \$

$$\text{Max } D = 1.25x + .4y + .67z$$

SUB TO

$42x + 21y + 35z \le 12000$ (total \$)

$21y \ge 2(42x)$ (ratio INTC and KO)

$35z \le .25(42x + 21y + 35z)$ (limit on MCD)

$x, y, z \ge 0$

5. Every day a baker has 44 units of flour, 26 units of sugar and 100 units of raisins. A loaf of raisin bread requires 2 units of flour, 1 unit of sugar and 2 units of raisins. A raisin cake requires 1 unit of flour, 1 unit of sugar and 5 units of raisins.

(a) Raisin bread sells for $2 per loaf and raisin cakes sell for $5 each. How many of each item should be made to maximize revenue? What is the maximum revenue? Is anything leftover? Fractional items are not allowed?

(b) If the price of raisin breads is increased to $3 per loaf, how many of each item should be made? What is the revenue?

(c) What range of prices are possible for the raisin cakes so that the solution remains at the point above?

(d) What is the range in value for the raisins ordered per day? What is the shadow price for raisins?

x = # of R.B.
y = # of R.C.
R = rev. in $

Max $R = 2x + 5y$

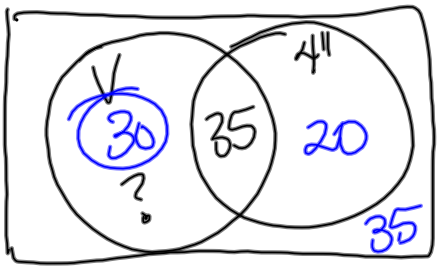
S.T.

$2x + y \leq 44$ (flour)
$1x + 1y \leq 26$ (sugar)
$2x + 5y \leq 100$ (raisins)
$x \geq 0,\ y \geq 0$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

SETS $85 = 65 + 55 - n(V \cap 4'') \Rightarrow n(V \cap 4'') = 35$

1. A survey of 120 plants at a nursery found that 65 of the plants were vegetables and 55 of the plants were in 4" pots. If 85 of the plants were vegetables or in 4" pots, how many vegetables were not in 4" pots? 30



$n(U) = 120$
$n(V) = 65$
$n(4'') = 55$
$n(V \cup 4'') = 85$

2. Let U = {x|x is a card in a standard deck of 52 cards} and
K = {x∈U | x is a king}
Q = {x∈U | x is a queen}
H = {x∈U | x is a heart}
Find KUQ, K∩Q, HUQ, H∩Q, $H^C \cup Q^C$

bring to lab

$K \cup Q = \{KD, KH, KS, KC, QH, QD, QS, QC\}$

$K \cap Q = \emptyset$

$H \cup Q = \{AH, 2H, \ldots JH, QH, KH, QD, QS, QC\}$ has 2^4 subsets

$H \cap Q = \{QH\}$

3. Let U = {a, b, c, d, e, f}, V = {a, b, f}, W = {b, c, d, e}. Determine if the following statements are true or false.

a) $\{b\} \in W$ F $\Rightarrow$ $b \in W$ or $\{b\} \subset W$ are true

F b) $V \cup W = \{a, b, b, c, d, e, f\}$ only 1 b

c) $\varnothing \subseteq V$ T

T d) V has 7 proper subsets

V has $2^3 = 8$ subsets. One of these is {a, b, f} so $8 - 1 = 7$ proper subsets

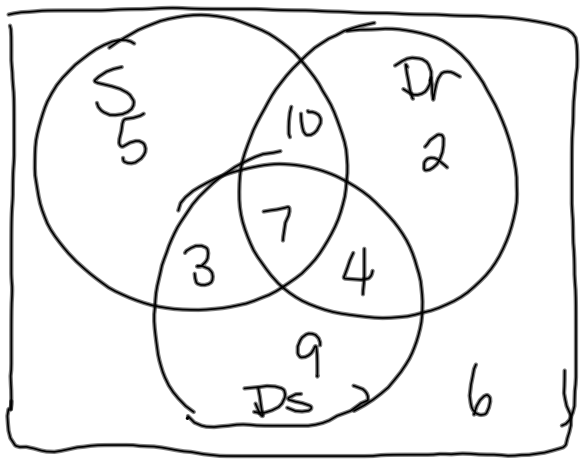
T e) $(A \cup A^c) = U$

$A \cap A^c = \emptyset$ T

4. At a party, it was noticed that
   - 7 guests brought a salad, a dessert and drinks
   - 17 guests brought exactly two items
   - 23 guest brought drinks
   - 21 guests did not bring a salad
   - 9 guests brought only a dessert
   - 11 guests did not bring a drink or a dessert
   - 10 guests brought only a salad and a drink
   - 11 guests brought a drink and a dessert

How many guests were at the party?

46



COUNTING

1. A PIN code has 3 letters. How many different PIN codes are possible if 3 of the same letter are not allowed (like AAA), but two of the same are allowed (AAB, ABA and BAA)?

$26 \cdot 26 \cdot 26 - 26 =$ _____

(26 Letter · 26 Letter · 26 Letter)

2. You are dealt a hand of 5 cards. How many ways can you be dealt at least 3 hearts?

3H 2H^C + 4H 1H^C + 5H 0H^C

$C(13,3)\ C(39,2) + C(13,4)\ C(39,1) + C(13,5)\ C(39,0)$

3H 2H^C 4H 1H^C 5H 0H^C

3. Three families are going to a basketball game together. The Smith family has 3 members, the Jones family has 4 members and the Garcia family has 6 members. How many ways can these 13 people be seated in a row if members of the same family sit together?

4. A mantel is being decorated for Spring. There are 8 decorations: 1 large bunny, 4 identical baby bunnies, a pair of identical candlesticks and a tulip plant. How many distinguishable ways can these decorations be arranged?

5. A bag has 5 pennies, 4 dimes and 6 quarters. Four coins are chosen at random from the bag

a. How many ways can the chosen coins be all quarters?

b. At least one penny

c. How many ways can the chosen coins have exactly 3 pennies or exactly 1 dime?

6. A box of 12 donuts will be purchased from a shop with 16 kinds of donuts. How many ways can the donuts be chosen?

BASIC PROBABILITY

1. A cup has one gold, one silver and one bronze coin in it. A single coin is chosen at random from the cup. How many events for this experiment contain a gold or silver coin?

2. A bowl has 2 red and 3 green apples. A sample of 2 is chosen at random from the bowl. How many outcomes are in the uniform sample space for this experiment?

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3. Explain why
$P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n) \leq P(A_1) + P(A_2) + P(A_3) + \cdots P(A_n)$.

4. Express $P(F \cap G^C)$ using $P(F)$, $P(G)$, and $P(F \cap G)$.

5. A class has 150 students and the maximum grade possible in this class is 100. Eleven students had a grade of 90 or more. Forty-one students had grades of 80 or more. Fifty-seven students had a grade that was greater than or equal to 60 but less than 70. Ten students had grades less than 60. If x represents a student's grade, organize this information in the probability distribution table