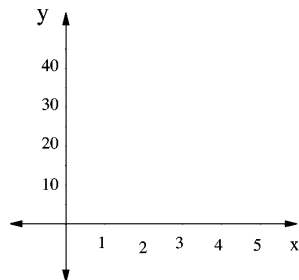# Chapter1:   Lines and Linear Modeling

1.1 The Cartesian Coordinate System

A properly labeled set of axes should look something like this:
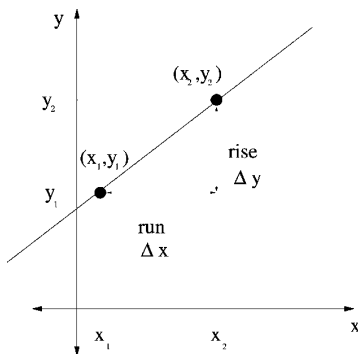

1.2 Straight Lines

A vertical line has NO SLOPE. All other lines have a slope given by the equation

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

The slope is a ratio of how y changes as x changes:

POINT-SLOPE form: $y - y_1 = m(x - x_1)$

SLOPE-INTERCEPT form: $y = mx + b$, b is the y-intercept

GENERAL form: $Ax + By + C = 0$

Example: A line has a slope of 2 and goes through the point (3, 4). What is the equation of the line?

Example: Find the intercepts for the line $y = 2x - 2$. Show these on a graph.

1.3 Linear Functions and Mathematical Models

A linear model is a model that is a linear function,
$$f(x) = y = mx + b$$
x is the independent variable (horizontal axis)
y the dependent variable (vertical axis).
The domain is the allowed values for the independent variable.

Depreciation

- An item has an initial value and a final (scrap) value.
- It is assumed that the value decreases linearly with time

Example: A car is purchased for $15,000 and is kept for 5 years and at the end of the 5 years it is worth $5000. Find an expression for the value of the car as a function of time and graph it. What is the car worth after 4 years? What is the rate of depreciation of the car?

Cost
- The cost of producing x items is written $C(x) = cx + F$
- The fixed costs, F, are those costs that are independent of the number of items produced.
- The variable costs are those costs that vary as the number of items produced. The slope c is the cost to make one more item.

Example: A company makes heaters. They find that the cost to make 10 heaters is $1500 and the cost to make 20 heaters is $1900. Find the cost equation and graph it.

Revenue

If x items are sold for s dollars each, the money brought in, or revenue, from the sale of these items is

$$R(x) = sx$$

Example: Example - What is the revenue from selling heaters if the heaters sell for $50 each?

Profit

The profit made from selling x items is $P(x)$ and it is the difference between the revenue (money in) and the cost (money out).

$$P(x) = R(x) - C(x)$$

Example: What is the profit equation from the heaters?

Demand

p is the price of an item. The demand is $D(x) = p$. This models the relationship between the price and the number of items purchased by the consumer.

Example: Say a store finds that it can sell 10 trench coats when the price is $180 each and that is sells 50 trench coats when the price is $100 each. Find the demand equation.

Supply

The supply function $S(x) = p$ models the relationship between the price of an item and how many are supplied to the market.

Example: A company manufactures trench coats. The company is not willing to sell trench coats unless it can get \$60 each. It will supply 10 trench coats if it can get \$80 each. Find the supply equation.

1.4 Intersection of Straight Lines

Equilibrium Point

Is there a price that will satisfy the consumer and the producer? Plot the supply and demand equations on the same graph. If the lines intersect, that will be at a price and number of items that the producer and consumer agree upon. It is called the *equilibrium point*.

The equilibrium point is (x_0, p_0).
     x_0 is the equilibrium quantity
     p_0 is the equilibrium price.

Example: What is the equilibrium point for the sale of trench coats?

Break-Even Analysis

Where the company's costs are equal to their revenue is the break-even point.

Example - What is the break-even point for the company making heaters?

1.5 The Method of Least Squares

The table below shows the demand for boxes of Lunchie treats where x is the number of boxes (in thousands) and y is the price in dollars.

x	11	16	21	27	32
y	4.00	3.15	3.25	3.00	2.75

Graph this in a scatter plot. What might the demand equation look like?

With linear regression we want to find a line that takes all of the data into account. The line should come as close as possible to all the data points. The line that has the smallest sum of the distance from all the points is the least squares (or regression) line. The equation of the line is found using your calculator or excel – do not use the formulas in the book!

The correlation coefficient, r measures how close the data points are to the line. The closer the value is to $|1|$, the better the linear fit is. If the value is near 0, the data is not very linear.

Example: Use linear regression to find the demand equation for Lunchie treats. How many boxes would be sold at a price of $3.75 each? If the company wants to see 14,000 boxes, what should the price be?
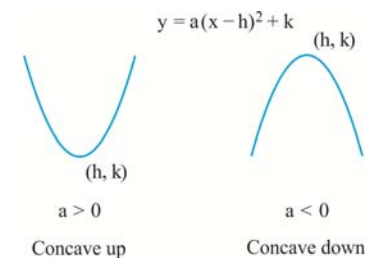
Quadratic Models

A quadratic is a polynomial of order 2:
$$q(x) = ax^2 + bx + c, \, a \neq 0.$$
Every quadratic function can be written in standard form:
$$y = a(x-h)^2 + k \text{ where } h = -\frac{b}{2a} \text{ and } k = c - \frac{b^2}{4a}$$

$y = a(x - h)^2 + k$

(h, k)

(h, k)

$a > 0$ $a < 0$

Concave up   Concave down

Example: What is the revenue from selling espressos if the demand equation for selling espressos is $p = -0.25 + 5$? Graph the revenue equation and interpret the result.