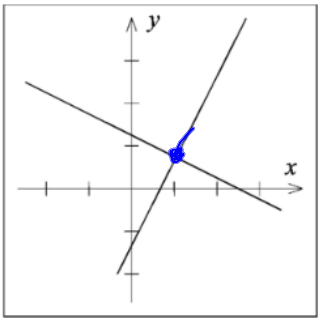
Chapter 2: Systems of Linear Equations and Matrices

2.1 Systems of Linear Equations: An Introduction

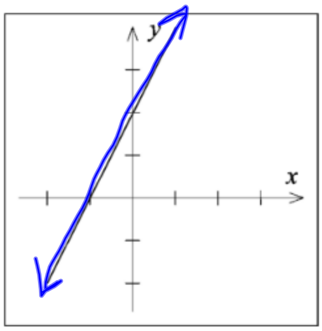
1. The two lines intersect.

Intersection point is the solution to the system of 2 eqns and 2 variables



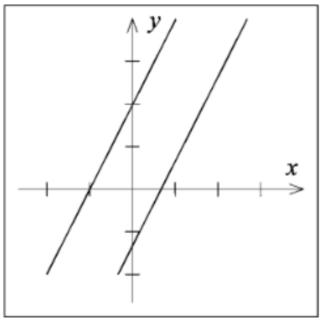
2. The two lines are the same.

The soln is the line



3. The two lines are parallel.

No solution

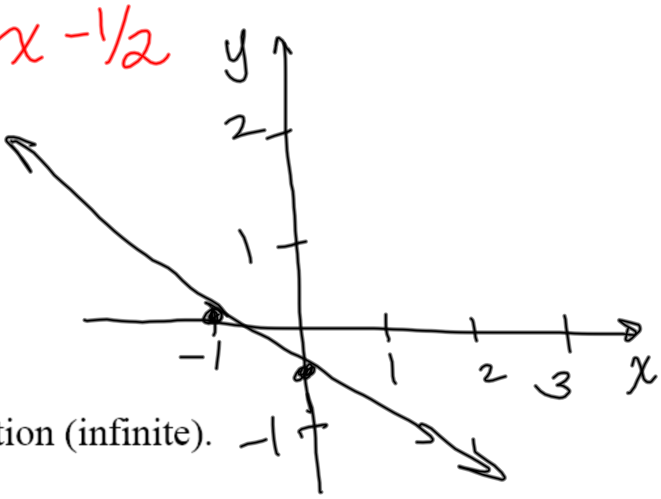


Example: Find the solution to the system of equations $\boxed{\begin{aligned} x+2y&=-1 \\ 2x+4y&=-2 \end{aligned}}$

$x=-2y-1$

$2(-2y-1)+4y=-2 \Rightarrow -2=-2 \Rightarrow 0=0$

Both are $y=-\frac{1}{2}x-\frac{1}{2}$



This system is called dependent.

The solution is a parametric solution (infinite).

We can consider y as a parameter,

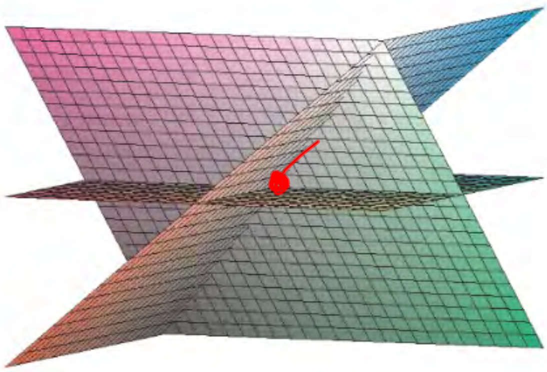
$(x,y)=(\underline{-2y-1}, y)$, y is any $\mathbb{R}$

$=(-2t-1, t)$, t is any $\mathbb{R}$

Particular solutions: $t=0 \Rightarrow (-1,0)$

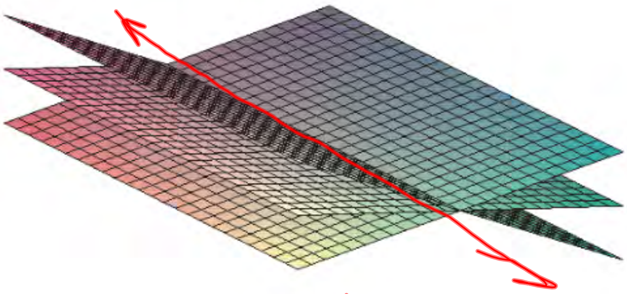
$t=1 \Rightarrow (-3,1)$

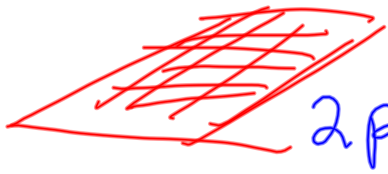
Example: Three Linear Equations with 3 Variables



(x, y, z)

ONE SOLUTION

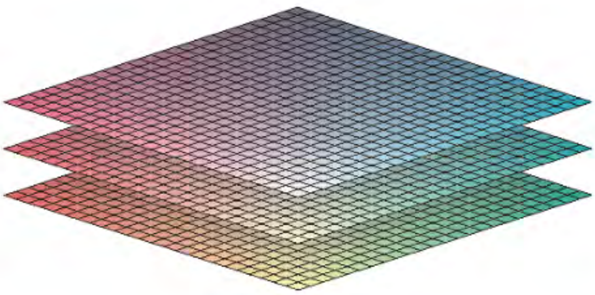




2 par

1 par

PARAMETRIC



NO SOLUTION

Formulating Linear Systems

An apartment complex is being developed that has small, large, and luxury apartments. The developer has decided that there will be a total of 192 apartments. The number of bigger apartments (large and luxury) will equal the number of small apartments. The number of small apartments will be three times the number of luxury apartments. How many apartments of each type will there be?

x = the number of small apartments

y = # of lg apt

z = # of lux apts

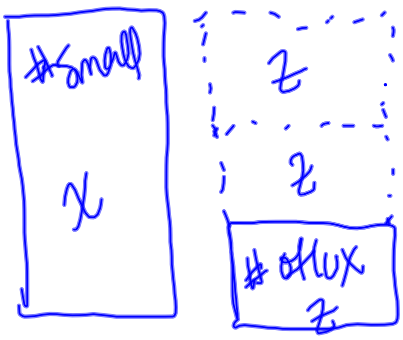
"The developer has decided that there will be a total of 192 apartments.

$$x + y + z = 192$$ (total apts)

"The number of bigger apartments (large and luxury) will equal the number of small apartments."

$$y + z = x$$ (lg apt = sm apt)

"The number of small apartments will be three times the number of luxury apartments."



$x = 3z$ (ratio of small to lux)

$$x = 3z$$

4

Zelda has $8,900 to invest. She decides to invest all of her money in three different funds. The PX Company costs $50 per share and pays dividends of $1.00 per share each year. The NY Company costs $90 per share and pays dividends of $2.00 per share each year. The LZ Company costs $40 per share and pays dividends of $1.60 per share per year. Zelda wants to invest half as much money in the LZ Company as in the NY Company and wants to earn $222 in dividends per year. How many shares of each company should Zelda buy to meet her goal?

x = # of sh of PX

y = # of sh of NY

z = # of sh of LZ

"The PX Company costs $50 per share and pays dividends of $1.00 per share each year." If you buy x shares in the PX Company,

How much does it cost? $ $50x$
How much do you earn in dividends? $ $1x$

"The NY Company costs $90 per share and pays dividends of $2.00 per share each year." If you buy y shares in the NY Company,

How much does it cost? $ $90y$
How much do you earn in dividends? $ $2y$

"The LZ Company costs $40 per share and pays $1.60 per share per year in dividends." If you buy z shares in the LZ Company,

How much does it cost? $ $40z$
How much do you earn in dividends? $ $1.6z$

Write an equation for the statement "Zelda has \$8,900 to invest."

$$50x + 90y + 40z = 8900$$ (total \$ invested)

Write an equation for the statement "Zelda wants to earn \$222 in dividends per year."

$$x + 2y + 1.6z = 222$$ (total div in \$)

"Zelda wants to invest half as much money in the LZ Company as in the NY Company."

$$90y = 2(40z)$$

(ratio of \$ in NY and LZ)

~~Put it all together for your answer~~:

Be sure to ① define variables

② write eqns

③ explain eqns (in)

2.2 Systems of Linear Equations: Unique Solutions

You may:

- Interchange any two of the equations.
- Multiply an equation by a non-zero constant.
- Add a multiple of one equation to another.

$$2x - y = 6$$
$$x + 2y = -2$$

$\rightsquigarrow$ $x = a$, $y = b$

AUGMENTED MATRIX form

$$\begin{array}{ccc|c} & x & y & = \ \# \\ & \left[\begin{array}{cc} 2 & -1 \\ 1 & 2 \end{array}\right. & & \left.\begin{array}{c} 6 \\ -2 \end{array}\right] \end{array}$$

"messy" one $\rightsquigarrow$

$$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$$

"clean" one

Rules are

- Interchange any of the rows equations.
- Multiply a row by a non-zero constant.
- Add a multiple of one row to another.

The goal is to get the augmented matrix into row-reduced echelon form (RREF).

What is RREF form?

1. All rows consisting entirely of zeros are at the bottom of the matrix
2. The first non-zero entry in any row is a 1 (called a leading 1)
3. In any two successive non-zero rows the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
4. If a column contains a leading 1, then the rest of the entries in that column are 0's.

PIVOT on an element – make the element a 1 and the rest of the entries in that column 0's.

Gauss-Jordan Method – Pivot until the augmented matrix is in RREF form.

$$\begin{aligned}2x - y &= 6\\ x + 2y &= -2\end{aligned} \Rightarrow \left[\begin{array}{cc|c} 2 & -1 & 6 \\ 1 & 2 & -2 \end{array}\right] \xrightarrow{GJ} \left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right] \Rightarrow \begin{aligned} x &= a \\ y &= b \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & -1 & 6 \\ 1 & 2 & -2 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 3 \\ 1 & 2 & -2 \end{array}\right] \xrightarrow{-1 * R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 3 \\ 0 & 2.5 & -5 \end{array}\right]$$

$-1 \quad +\frac{1}{2} \quad -3$

$$\xrightarrow{\frac{1}{2.5} * R_2 \to R_2} \begin{array}{ccc} 0 & \frac{1}{2} & -1 \\ \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 3 \\ 0 & 1 & -2 \end{array}\right] \xrightarrow{\frac{1}{2}R_2 + R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -2 \end{array}\right] \Rightarrow \begin{aligned} x &= 2 \\ y &= -2 \end{aligned}$$

$$\begin{aligned} x + y + z &= 192 \\ y + z &= x \\ x &= 3z \end{aligned} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 192 \\ -1 & 1 & 1 & 0 \\ 1 & 0 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 96 \\ 0 & 1 & 0 & 64 \\ 0 & 0 & 1 & 32 \end{array}\right] \Rightarrow \begin{aligned} x &= 96 \\ y &= 64 \\ z &= 32 \end{aligned}$$

There are 96 small, 64 large and 32 luxury apts.