

Example

A class of 12 students will divide into 3 teams of 4. How many ways can this be done?

$$\frac{C(12,4)}{\text{team 1}} \cdot \frac{C(8,4)}{\text{team 2}} \cdot \frac{C(4,4)}{\text{team 3}}$$

Example

A school is putting together a committee. The committee will have a chair and an assistant chair chosen from a group of 10 teachers, two parents chosen from a group of 15 parents and two students chosen from a group of 20 students. How many different committees are possible?

$$\frac{10 \cdot 9}{C} \cdot \frac{C(15,2)}{AC} \cdot \frac{C(20,2)}{2 \text{ Parents } 2 \text{ Students}}$$

Example

You are dealt a hand of four cards from a well-shuffled standard deck of 52 cards.

(a) How many ways can you be dealt at least 3 spades?

$$3S 1S^c + 4S 0S^c$$

$$\frac{C(13,3)}{3S} \frac{C(39,1)}{1S^c} + \frac{C(13,4)}{4S} \frac{C(39,0)}{0S^c}$$

"at most" or "or" $\Rightarrow +$
"and" $\Rightarrow -$

(b) How many ways can you be dealt exactly two diamonds or exactly two clubs?

$$2D 2D^c + 2C 2C^c - 2D 2C$$

$$\frac{C(13,2)}{2D} \frac{C(39,2)}{2D^c} + \frac{C(13,2)}{2C} \frac{C(39,2)}{2C^c} - \frac{C(13,2)}{2D} \frac{C(13,2)}{2C}$$

neither

Example

How many different "words" can be made from the letters in MISSISSIPPI?

$$\frac{11!}{\left(\begin{matrix} 4! & 4! & 2! & 1! \\ \text{arr} & \text{I} & \text{P} & \text{M} \\ \text{the} & & & \end{matrix} \right)} = 34,650$$

If you are arranging n items some of which are identical (n_1 of type 1, n_2 of type 2, etc)
 # of dist. arrangements is $\frac{n!}{(n_1!)(n_2!) \dots (n_i!)}$

Example

Seven children stop at a restaurant where they have a choice of a cheeseburger, a hot dog, pizza or a burrito. How many different purchases are possible?

could have

<u>c c c c h p b</u>	* * * * * * *
<u>h h p p p b b</u>	* * * * * * * *
<u>c h h h h h</u>	* * * * * *
<u>c c c c c b</u>	* * * * * *

7 items → * Star
 4 groups → | bars

total of 7+3 things ⇒ $10! / (7! \times 3!)$

How many subsets? for a set with 1 element

$\{A\} : \{A\}, \emptyset$ 2 subsets

$\{A, B\} : \{A, B\}, \{A\}, \{B\}, \emptyset$ 4 subsets

$\{A, B, C\} : \{A, B, C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A\}, \{B\}, \{C\}, \emptyset$ 8 subsets

set with n elements has 2^n subsets

$$\frac{2}{1} \cdot \frac{2}{2} \cdot \frac{2}{3} \dots \frac{2}{n} = 2^n$$