CHAPTER 7: PROBABILITY

7.1: Experiments, Sample Spaces and Events

An ***experiment*** is an activity with an observable result. Tossing coins, rolling dice and choosing cards are all probability experiments.

The result of the experiment is called the ***outcome*** or ***sample point***. The set of all outcomes or sample points is called the ***sample space*** of the experiment.

Example
What is the sample space for flipping a fair coin? Rolling a 6-sided die?

$$S = \{H, T\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

An ***event*** is a subset of a sample space. That is, an event can contain one or more outcomes that are in the sample space.

Example

→ subsets

What are all possible events for the experiment of flipping a fair coin?

$$\emptyset, \{H\}, \{T\}, \{H, T\}$$

$\subseteq$ subset

$\subset$ proper subset

$\in$ element of

$\{1\} \subset \{1, 2, 3, 4, 5, 6\}$

$1 \in \{1, 2, 3, 4, 5, 6\}$

Example
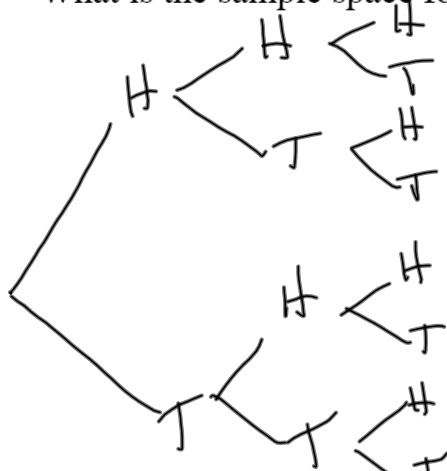How many events are possible when a six-sided die is rolled?

$$2^6 = 64$$

$$\emptyset, \{1\}, \ldots$$

$$\{1, 2, 3, 4, 6\}, \{1, 2, 3, 4, 5, 6\}$$

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Example $n(S) = 2 \cdot 2 \cdot 2 = 8$

What is the sample space for flipping a fair coin three times?

$S = \{HHH, HHT, HTH,$
$HTT, THH, THT,$
$TTH, TTT\}$

2^8 events

Find the event E where $\dot{E} = \{x | x$ has exactly one head$\}$

$E = \{HTT, THT, TTH\}$

Find the event *E* where $E = \{x | x$ has two(or more heads$\}$

$E = \{HHT, HTH, THH, HHH\}$

Find the event *E* where $E = \{x | x$ has more than 3 heads$\}$

$\emptyset$

A sample space in which each of the outcomes has the same chance of occurring is called a UNIFORM SAMPLE SPACE.

Example

A bowl contains the letters AGGIES. How many outcomes are in the uniform sample space?

$S = \{A, G, I, E, S\}$ ← NOT UNIFORM

$S = \{A, G, G, I, E, S\}$ $n(S) = 6$

What is the uniform sample space for rolling two fair six-sided dice?

<div style="color:green">

1~1 2~1 3~1 4~1 5~1 6~1
1~2 2~2 3~2 4~2 5~2 6~2
1~3 2~3 3~3 4~3 5~3 6~3
1~4 2~4 3~4 4~4 5~4 6~4
1~5 2~5 3~5 4~5 5~5 6~5
1~6 2~6 3~6 4~6 5~6 6~6

</div>

$$n(S) = 36$$

These sample spaces have all been finite. That is, we can list all the elements. An infinite sample space has to be described; you can't list all the elements:

Example
What is the sample space for the time spent working on a homework set?

$$S = \{ t \mid t \geq 0,\ t \text{ in minutes} \}$$

Describe the event of spending between one and two hours on a homework set.

$$E = \{ t \mid 60 < t < 120 \}$$