CHAPTER 7: PROBABILITY

7.1: Experiments, Sample Spaces and Events

An ***experiment*** is an activity with an observable result. Tossing coins, rolling dice and choosing cards are all probability experiments.

The result of the experiment is called the ***outcome*** or ***sample point***. The set of all outcomes or sample points is called the ***sample space*** of the experiment.

Example

What is the sample space for flipping a fair coin? Rolling a 6-sided die?

An ***event*** is a subset of a sample space. That is, an event can contain one or more outcomes that are in the sample space.

Example

What are all possible events for the experiment of flipping a fair coin?

Example

How many events are possible when a six-sided die is rolled?

Example

What is the sample space for flipping a fair coin three times?

Find the event E where $E = \{x | x \text{ has exactly one head}\}$

Find the event E where $E = \{x | x \text{ has two or more heads}\}$

Find the event E where $E = \{x | x \text{ has more than 3 heads}\}$

A sample space in which each of the outcomes has the same chance of occurring is called a UNIFORM SAMPLE SPACE.

Example

A bowl contains the letters AGGIES. How many outcomes are in the uniform sample space?

What is the uniform sample space for rolling two fair six-sided dice?

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

These sample spaces have all been finite. That is, we can list all the elements. An infinite sample space has to be described; you can't list all the elements:

Example
What is the sample space for the time spent working on a homework set?

Describe the event of spending between one and two hours on a homework set.

7.2 Definition of Probability

The probability of an event, $P(E)$ is a number between 0 and 1, inclusive. If $P(E)=0$, then the event E is impossible. If $P(E)=1$, then the event E is certain.

The ***theoretical probability*** of an event E occurring is based on the sample space S having equally likely outcomes. Then probability of the event E occurring is

$$P(E)=\frac{\text{number of outcomes in event E}}{\text{number of outcomes in the sample space}}=\frac{n(E)}{n(S)}$$

Example
Consider flipping a fair coin three times. Find the following probabilities:

(a) Exactly one head is seen.

(b) Two or more heads are seen.

(c) More than 3 heads are seen.

We can also calculate the ***empirical probability*** of an event by doing an experiment many times. Roll a six sided die and count the number of times a 1 is observed.

# of tosses (m)	# of 1's rolled (n)	relative frequency (n/m)

Consider the uniform sample space $S=\{s_1,s_2, ...,s_n\}$, with *n* outcomes. The *n* events that contain a single outcome, $\{s_1\}$, $\{s_2\}$ . . . $\{s_n\}$ are called ***simple*** events.

Events that can't occur at the same time are called ***mutually exclusive***. Note that the simple events are mutually exclusive.

A ***probability distribution table*** has the following properties:

1. Each of the entries is mutually exclusive with all other entries
2. The sum of the probabilities is 1

PROBABILITY DISTRIBUTION TABLE:

Event	probability

<u>*Example*</u>

Find the probability distribution table for the number of heads when a coin is tossed 3 times.

What is the probability of 2 or more heads?

Suppose the instructor of a class polled the students about the number of hours spent per week studying math during the previous week. The results were 69 students studied two hours or less, 128 students studied more than two hours but 4 or less hours, 68 students studied more than 4 hours but less than or equal to 6 hours, 30 students studied more than 6 hours but less than or equal to 8 hours and 14 students studied more than 8 hours.

Arrange this information into a PDT and find the probability that a student studied more than 4 hours per week

What is the probability of rolling a sum 2 or a sum of 12 using two fair die?

1~1 2~1 3~1 4~1 5~1 6~1
1~2 2~2 3~2 4~2 5~2 6~2
1~3 2~3 3~3 4~3 5~3 6~3
1~4 2~4 3~4 4~4 5~4 6~4
1~5 2~5 3~5 4~5 5~5 6~5
1~6 2~6 3~6 4~6 5~6 6~6

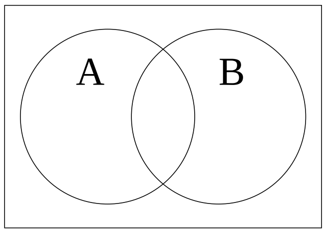
What is the probability of rolling a sum of 7?

1~1 2~1 3~1 4~1 5~1 6~1
1~2 2~2 3~2 4~2 5~2 6~2
1~3 2~3 3~3 4~3 5~3 6~3
1~4 2~4 3~4 4~4 5~4 6~4
1~5 2~5 3~5 4~5 5~5 6~5
1~6 2~6 3~6 4~6 5~6 6~6

7.3 Rules of Probability

If event *A* and event *B* are mutually exclusive then

In general, *A* and *B* have some outcomes in common so we have the union rule for probability:



Example

E={x|x is a sum of 7} =

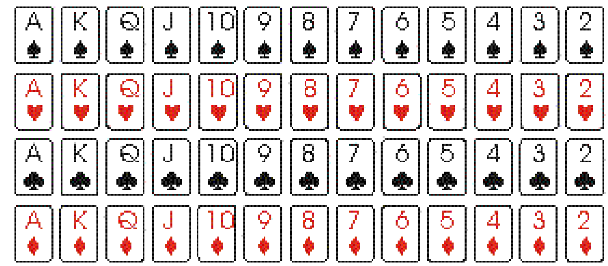
and

F={x|x is a 6 on the green die} =

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

What is the probability that you have a sum of 7 OR a 6 on the green die?

A standard deck of 52 cards has 4 suits, each with 13 cards. The suits are spades, ♠, hearts, ♥, clubs, ♣ ,and diamonds, ♦. The cards in each suit are numbered from Ace, King, Queen, Jack, ten down to 2.



Example

If a single card is drawn from a standard deck of cards, what are the probabilities of

a) a 9 or a 10?

b) a black card or a 3?

Example

A survey gave the following results: 45% of the people surveyed drank diet drinks (D) and 25% drank diet drinks and exercised ($D \cap E$) and 24% did not exercise and did not drink diet drinks ($D^c \cap E^c$). Find the probability that:

a) a person does not drink diet drinks (D^c).

b) does not exercise and drinks diet drinks ($E^c \cap D$).

c) exercises and does not drink diet drinks ($E \cap D^c$).

7.4 Use of Counting Techniques in Probability

Let S be a uniform sample space and E be any event in S. Then

$$P(E) = \frac{\text{number of outcomes in event E}}{\text{number of outcomes in the Sample Space}} = \frac{n(E)}{n(S)}$$

Example

Suppose we have a jar with 8 blue and 6 green marbles. What is the probability that in a sample of 2, both will be blue?

What is the probability there is at least one blue marble?

Find the probability distribution table for the number of blue marbles in the sample of 2 marbles:

Example

A stack of 100 copies has 3 defective papers. What is the probability that in a sample of 10 there will be no defective papers?

Example

A student takes a true/false test with 5 questions by guessing (choose answer at random). Write a probability distribution table for the number of correct answers.

7.5 Conditional Probability and Independent Events

A survey is done of people making purchases at a gas station. Most people buy gas (Event A) or a drink (Event B).

	buy drink (B)	no drink (B^c)	total
buy gas (A)			
no gas (A^c)			
total			

What is the probability that a person bought gas and a drink?

What the probability that *a person who buys a drink* also buys gas? In other words, **given** that a person *bought a drink* (B), what is the probability that they bought gas (A)?

NOTATION: P(A|B) = the probability of A given B

The ***conditional probability*** of event *E* given event *F* is

$$P(E\mid F)=\frac{n(E\cap F)}{n(F)}=\frac{n(E\cap F)/n(S)}{n(F)/n(S)}=\frac{P(E\cap F)}{P(F)}$$

What is the probability that *a person who buys gas* also buys a drink?

PRODUCT RULE: $P(E \cap F) = P(E)P(F|E)$

TREE DIAGRAMS

At a party, 1/3 of the guests are women. 75% of the women wore sandals and 25% of the men wore sandals. What is the probability that a person chosen at random at the party is a man wearing sandals, $P(M \cap S)$? What is the probability that a randomly chosen guest is wearing sandals?

A finite stochastic process is one in which the next stage of the process depends on the state you are in for the previous stage.

Consider drawing 3 cards from a standard deck of 52 cards without replacement.
(a) What is the probability that the three cards are hearts?
(b) What is the probability that the third card drawn is a heart given the first two cards are hearts?

INDEPENDENT EVENTS: P(E|F) = P(E)

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = P(E) \quad \Rightarrow$$

$$P(E \cap F) = P(E) \cdot P(F) \text{ } iff \text{ } E \text{ and } F \text{ are independent}$$

Example
A certain part on airplane has a 1% chance of failure. If they carry two back-ups to this part, what is the probability that they will all fail?

Example
A medical experiment showed the probability that a new medicine was effective was 0.75, the probability of a certain side effect was 0.4 and the probability for both occurring is 0.3. Are these events independent?

7.6 Bayes' Theorem

Example
We are to choose a marble from a cup or a bowl. We need to flip a coin to decide to choose from the cup or the bowl. The bowl contains 1 red and 2 green marbles. The cup contains 3 red and 2 green marbles. What is the probability that a marble came from the cup given that it is red?

Example
A company makes the components for a product at a central location. These components are shipped to three plants, Alpha, Beta, and Gamma, for assembly into a final product. The percentages of the product assembled by the three plants are, respectively, 50%, 20%, and 30%. The percentages of defective products coming from these three plants are, respectively, 1%, 2%, and 3%. Given a defective product, what is the probability it was assembled at plant Alpha?