

FINANCE

① $Pe^{rt} = 600 e^{(.086 \times 44/12)} = 822.42968... \rightarrow \822.43

$F = P(1 + rt) = 600(1 + .086 \times 44/12) = \789.20

② a) $N = ?$ $PMT = 100$ $\rightarrow N = 81.27, \dots$
 $I = 24$ $FV = 0$ $\Rightarrow 82 \text{ months}$
 $PV = 4000$ $P/Y = 12$

b) $100 \times 82 = \$8200$, $8200 - 4000 = \$4200$ in interest

c) $4000 \times .24/12 = \$80$, \$20 to pay off balance

d) Owe $4000 - 20 = 3980$, Interest = $3980 \times .24/12 = \$79.6$

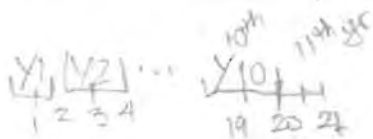
e) $N = 82 - 24 = 58$ PMT LEFT $PMT = -100$
 $I = 24$ $FV = 0$ $\rightarrow \$3414.52$
 $PV = ?$ (what you owe) $P/Y = 12$

③ $N = 25 \times 12$ $PMT = 1600$ $\rightarrow \$248,330.98$
 $I = 6$ $FV = 0$
 $PV = ?$ $P/Y = 12$

④ $N = 19$ $PMT = 1,000,000$ $\rightarrow \$9,267,720.22$
 $I = 8.5$ $FV = 0$
 $PV = ?$ $P/Y = 1$

⑤ $N = 10 \times 2$ $PMT = 3000$
 $I = 9$ $FV = ? \rightarrow \$94,114.27 \times \frac{.09}{2} = \4235.14 interest
 $PV = 0$ $P/Y = 2$ 1st 1/2 of 11th yr

$N = 20 \times 2$ $PMT = 3000$
 $I = 9$ $FV = ? \rightarrow \$321,090.97 \times \frac{.09}{2} = \$14,449.09$ interest
 $PV = 0$ $P/Y = 2$ 1st 1/2 of 21st yr



BONUS START WITH \$10,000 market,
 $N = 20$ $PMT = 3000$
 $I = 9$ $FV = ? \rightarrow \$118,231.41$
 $PV = 10000$ $P/Y = 2$

⑥ $Eff(2.6, 4) = 2.625\%$ ← Better deal

$Eff(2.58, 365) = 2.613\%$

⑦ $N=2$ $PMT=0$ $I = 5.409\%$
 $I=?$ $FV = -22222$
 $PV = 20000$ $P/Y = 1$ switch to lose $\$ \Rightarrow -5.131\%$

⑧ $\$220,000 \times .2 = \$44,000$ down payment
 \Rightarrow finance $220,000 - 44,000 = \$176,000$

a) $N=40 \times 12 = 480$ $PMT=? \rightarrow \$1005.43$
 $I = 6.3$ $FV = 0$ $(1005.43) \times 480 - 176,000$
 $PV = 176,000$ $P/Y = 12$ $= \$306,066.40$ in interest

b) $176,000 \times 0.063/12 = \924 in interest
 $1005.43 - 924 = \$81.43$ to pay off loan

c) $N = 480 - 10 \times 12 = 360$ PMT remaining $PMT = 1005.43$
 $I = 6.3$ $FV = 0$
 $PV = ?$, what you still owe $P/Y = 12$

\Rightarrow owe bank $162,435.52$ (Paid off $13,564.48$)
 $EQ = 220,000 - 162,435.52 = \$57,564.48$

d) $N = 360 \rightarrow PMT = \1089.39 , INT PAID = $\$216,180.40$

e) $N = 240 \rightarrow PMT = \1291.57 , INT PAID = $\$133,976.80$

9. $N = 51 \times 365$ $PMT = 0$
 $I = 4$ $FV = ? \rightarrow \$384.49$
 $PV = 50$ $P/Y = 365$

⑩ You have $\$500,000$ saved for retirement. An account pays 3% annual interest comp monthly. You withdraw $\$2000/mo$. How much after 10 years? Note $500,000 \times .03/12 = \1250
 $N = 120$ $PMT = 2000$ $\rightarrow \$395,194$
 $I = 3$ $FV = ?$
 $PV = 500,000$ $P/Y = 12$

SYSTEMS OF LINEAR EQNS

$$\textcircled{1} \begin{array}{c} x \quad y \quad z \quad w \\ \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{array} \quad \begin{array}{l} x - 2z = 0 \\ y + z + w = -3 \\ 0 = 0 \end{array} \quad \begin{array}{l} (x, y, z, w) = (2z, -3 - z - w, z, w) \\ t \text{ and } s \text{ any } \mathbb{R} \end{array}$$

- $\textcircled{2}$ (A) $s=0, t=0$ so $(-7, 1, 0, 0)$ is correct
 (B) $s=0, t=1$ so $(3(1)-7, 1, 0, 1) = (-4, 1, 0, 1)$ is correct
 (C) $s=1, t=0$ so $(-7, -2+1, 1, 0) = (-7, -1, 1, 0)$ NOT correct
 (D) $s=1, t=1$ so $(-4, -1, 1, 1)$ is correct

$\textcircled{3}$ $x = \# \text{ of carts}$ $9000x + 27000y + 27000z = 162,000$
 $y = \# \text{ of vans}$ $3x + 8y + 7z = 48$
 $z = \# \text{ of minivans}$

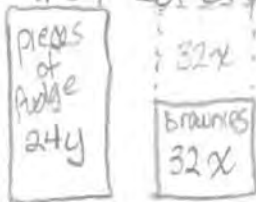
$$\left[\begin{array}{ccc|c} 9 & 27 & 27 & 162 \\ 3 & 8 & 7 & 48 \end{array} \right] \xrightarrow{\text{ref}} \begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 6 \end{array} \right] \end{array} \quad \begin{array}{l} x - 3z = 0 \\ y + 2z = 6 \end{array}$$

$$(x, y, z) = (3z, 6 - 2z, z) \quad t = \# \text{ of minivans}$$

$t=0 \Rightarrow (0, 6, 0)$ Buy 0 carts, 6 vans and 0 m-vans
 $t=1 \Rightarrow (3, 4, 1)$ Buy 3 carts, 4 vans and 1 m-van
 $t=2 \Rightarrow (6, 2, 2)$ Buy 6 carts, 2 vans and 2 m-vans
 $t=3 \Rightarrow (9, 0, 3)$ " 9 " 0 " 3 "

$\textcircled{2}$ Buy $t=0, 1, 2, 3$ minivans, $3t$ carts and $6-2t$ vans

$\textcircled{4}$ $x = \# \text{ of batches of brownies} \Rightarrow x + 1.5y = 15 \text{ lb nuts}$
 $y = \# \text{ of batches of fudge} \Rightarrow 24y = 2(32x)$



$\textcircled{2}$ $x = \# \text{ of brownies}$ $\Rightarrow (x/32) + 1.5(y/24) = 15 \text{ lbs nuts}$
 $y = \# \text{ of pieces of fudge}$ $y = 2x$

$$\textcircled{5} \quad \begin{cases} 6x + 0y + z = 5 \\ 9x + 1.5y + 0z = 8 \\ 3x + 18y + 0z = 8.5 \end{cases} \rightarrow \left[\begin{array}{ccc|c} 6 & 0 & 1 & 5 \\ 9 & 1.5 & 0 & 8 \\ 3 & 18 & 0 & 8.5 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{cases} x = 5/6 \\ y = 1/3 \\ z = 0 \end{cases}$$

$$\begin{cases} 6x + 9y + 0z = 5 \\ 9x + 0y + z = 8 \\ 3x + 4.5y + 0z = 9 \end{cases} \rightarrow \left[\begin{array}{ccc|c} 6 & 9 & 0 & 5 \\ 9 & 0 & 1 & 8 \\ 3 & 4.5 & 0 & 9 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 1/9 & 0 \\ 0 & 1 & -2/27 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \begin{matrix} \text{NO} \\ \text{SOLN} \end{matrix}$$

$\textcircled{6}$ $x = \# \text{ lb Columbian coffee}$

lb
col
x

+3
lb K
y

$y = \# \text{ lb Kona coffee}$

$z = \# \text{ lb Blue mt coffee}$

$$\begin{cases} x + y + z = 50 \\ x = y + 3 \\ 8x + 90y + 15z = 50 \times 11.70 \end{cases} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 1 & -1 & 0 & 3 \\ 8 & 10 & 15 & 585 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 23 \end{array} \right] \quad \begin{cases} x = 15 \\ y = 12 \\ z = 23 \end{cases}$$

Use 15 lb of Columbian coffee, 12 lb of Kona coffee and 23 lb of Blue mt coffee

Matrices #6: $X = (I - A)^{-1} \cdot D$. Store A as $[A]$, $D = 1500, 6500, 4000$ as a 3×1 matrix in $[D]$. Then $X = (\text{identity}(3) - [A])^{-1} \cdot [D] = 12694, 16127, 10189$ as a 3×1 matrix. So produce \$12,694 of stone, \$16,127 of farming, and \$10,189 of hunting. The entry in a_{12} says to make one unit of farming requires 0.3 units of stone.

MATRICES

$$\textcircled{1} A \cdot B = \begin{matrix} & P & Q \\ T & \begin{bmatrix} & \\ & \end{bmatrix} \\ S & \begin{bmatrix} & \\ & \end{bmatrix} \end{matrix} \begin{matrix} R & T & S \\ M & & \end{matrix} \text{ meaningless}$$

$$B \cdot A = \begin{matrix} T & S \\ R & \begin{bmatrix} .5 & .8 \\ .5 & .2 \end{bmatrix} \\ M & \begin{bmatrix} .5 & .2 \end{bmatrix} \end{matrix} \begin{matrix} P & Q \\ \begin{bmatrix} 60 & 75 \\ 45 & 50 \end{bmatrix} \\ S & \begin{bmatrix} .5 & .2 \end{bmatrix} \end{matrix} = BA = \begin{matrix} P & Q \\ R & \\ M & \end{matrix}$$

$$(BA)_{11} = \frac{.5 \times 60}{\text{Tiger brand road bikes at Paddington}} + \frac{.8 \times 45}{\text{snake brand road bikes in Paddington}} = \text{total road bikes at Paddington}$$

\Rightarrow Answer (B), # of Road & mtn bikes at each store

$$\textcircled{2} 2X + D = XB \Rightarrow D = XB - X \cdot 2I = X(B - 2I)$$

$$D(B - 2I)^{-1} = X(B - 2I)(B - 2I)^{-1} = X$$

$$\textcircled{3} \begin{bmatrix} a & -a \\ 2 & d \end{bmatrix}^T + 4 \begin{bmatrix} 1 & b \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & 2 \\ -2 & d \end{bmatrix} + \begin{bmatrix} 4 & 4b \\ 4c & 0 \end{bmatrix} = \begin{bmatrix} a+4 & 2+4b \\ -2+4c & d+0 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -6 & -3 \end{bmatrix}$$

$$\Rightarrow a+4=2 \Rightarrow a=-2 \qquad 2+4b=4 \Rightarrow b=1/2$$

$$-2+4c=-6 \Rightarrow c=-1 \qquad d=-3$$

$$\textcircled{4} G_1 \text{ is } 1 \times 3, FG_1 \text{ is } (3 \times 3)(1 \times 3) \text{ NOT POSSIBLE}$$

$$G_1 \times F \text{ (1} \times 3)(3 \times 3) \text{ \& } \begin{matrix} B & R & S \\ A & \begin{pmatrix} B & R & S \\ B \\ C \end{pmatrix} \end{matrix} \text{ (NO)}$$

$$G_2 \text{ } 3 \times 1 \quad FG_2 = \begin{matrix} B & R & S \\ A & \begin{pmatrix} B & R & S \\ B \\ C \end{pmatrix} \end{matrix} \begin{matrix} B \\ R \\ S \end{matrix} \checkmark$$

$$\textcircled{5} \text{ a) DNE} \quad \text{b) } 6 \times 6 \quad \text{c) DNE b/c } C \text{ is singular}$$

$$\text{d) } (7 \times 7)(7 \times 6) \text{ is } 7 \times 6 \quad \text{e) } (6 \times 6)(6 \times 7) \text{ is } 6 \times 7$$