Chapter F Finance

Section F.1 Simple Interest and Discount

Simple interest – interest computed as a percentage of the principal

Simple interest earned: $I = Prt$

I=interest earned
P=principal
r=interest rate (decimal form)
t=time period (in years)

$3\frac{1}{3} \neq 3.33$

Future value: $F = P + Prt = P(1 + rt)$

Example
If a bank loans $525 to an individual for 3½ years at 7.25% simple interest, what will be the amount repaid on the loan?

$F = 525(1+0.0725 \times 3.5) = 658.21875$

$\boxed{\$658.22}$

Example
If a $1000 deposit grows in value to $1024 after 9 months, what is the simple interest rate that is earned?

$I = 1024 - 1000 = 24 = 1000 * r * 9/12 = 750r$

$r = \frac{24}{750} = 0.032$ or 3.2%

Example

How much should be place in an account that pays simple interest of 4% so that the value of the account after 18 months is $3000?

$F = P(1+rt)$

$3000 = P(1+.04 \times 1.5) = 1.06 \cdot P$

$P = \frac{3000}{1.06} = 2830.188679 \rightarrow \boxed{\$2830.19}$

Example

You have an account with $\boxed{\$500}$ ×2 that pays 5% simple interest. How long until your account doubles in value?

$F = P(1+rt)$

$\frac{1000}{500} = \frac{500}{500}(1+.05t) \Rightarrow 2 = 1+.05t \Rightarrow 1 = .05t$

$-1 \quad -1$

$t = \frac{1}{.05} = \boxed{20 \text{ years}}$

When a loan is ***discounted***, the interest owed is deducted when the loan is made. The interest deducted is called the ***discount*** (D), the amount the borrower actually receives is called the ***proceeds*** (P) and the amount to be repaid is called the ***maturity value*** (M). So

$$D = Mrt \text{ and } P = M - D$$

Where t is in years and r is the annual simple interest rate.

Example

A borrower gets a loan in which she agrees to pay the bank $\overbrace{\$4000}^{M}$ in 8 months at 6% simple discount. What is the discount and what are the proceeds?

$D = 4000 \times .06 \times 8/12 = \boxed{\$160 = D}$

$P = 4000 - 160 = \boxed{\$3840 = P}$

Example

If a borrower wants to have proceeds of \$4000 on a discounted loan for 8 months at 6%, what is the maturity value of the loan?

$P = 4000 = M - D = \boxed{M - M \cdot (.06)(8/12) = 4000}$

$4000 = M(1 - .06 \times 8/12) = .96M$

$M = \frac{4000}{.96} = \boxed{\$4166.67 = M}$

$I = P \cdot r \cdot t$

Example

A borrower gets a loan in which she agrees to pay the bank \$4000 in 8 months at 6% simple discount. What is the effective interest rate?

$I = 160 = 3840 * r_{eff} * 8/12$

$r_{eff} = \frac{160}{3840 * 8/12} = .0625$ or $\boxed{6.25\%}$

The effective rate of interest on a discounted loan of length t years with a discount rate of r is $r_{eff} = \frac{r}{1 - rt}$

Section F.2 Compound Interest

When interest is paid periodically and the interest earns interest, we have compound interest.

$F = P(1+rt) \rightarrow$ SIMPLE $F = 100(1+.1\times4) = \$140$

Example

You deposit $100 into an account that pays 10% annual interest that is compounded annually. How much is in the account after 4 years?

after 1st year : $F_1 = 100(1+.1\times1) = \boxed{100*(1.1) = 110}$

after 2nd year : $F_2 = 110(1+.1\times1) = 110(1.1) = 100(1.1)(1.1) = \boxed{100(1.1)^2 = 121}$

3rd $F_3 = 121(1.1) = 100(1.1)^2\cdot(1.1) = 100(1.1)^3 = 133.10$

$F_4 = 133.10(1.1) = 100(1.1)^4 = 146.41$

N= 4
I= 10
PV= 100
PMT= 0
FV= ?
P/Y= 1

\$146.41

$$F = P\left(1+\frac{r}{m}\right)^{mt}$$

mt: # of times it's cmp

$\frac{r}{m}$ = interest earned in 1 period

where m is the number of compounding periods per year and r is the annual interest rate as a decimal.

$\frac{.06}{12} = .005$ 0.5% per month

Example

You deposit $300 into an account that pays 6% annual interest compounded monthly. How much is in the account after 3 months?

$F_1 = 300\left(1+\frac{.06}{12}\right) = 300(1.005) = 301.05$

$F_2 = 301.05(1.005) = 300(1.005)^2 = 303.0025$

$F_3 = 300(1.005)^3 = \boxed{\$304.52}$