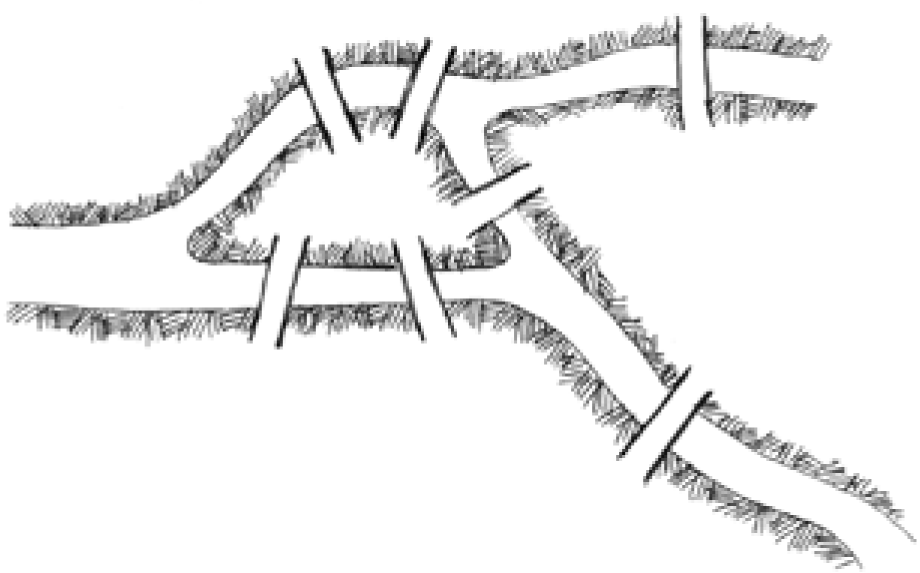
CHAPTER 1: URBAN SERVICES

Consider three interesting problems:

- What is the fewest number of colors needed to color a map?
- Is there a path over a series of streets [or bridges] that you can cross every street exactly once and end up where you started? Think mail delivery or trash pickup.
- What is the shortest route between a series of destinations? Think traveling salesman.

1.1 Euler Circuits

Here is a map of the bridges of Konigsberg:



Can you cross each bridge exactly once and return to your starting spot?

A ***graph*** is a collection of one or more points [***vertices***] and the connections between them [***edges***]

Example

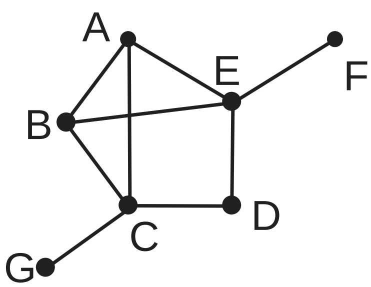
Draw a graph of the interstate highway connections between Oklahoma City, Dallas, Shreveport, Austin, San Antonio, Houston and Baton Rouge.

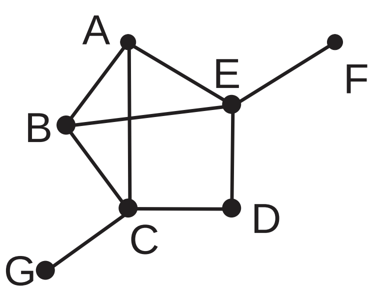
Two vertices are ***adjacent*** if they are connected by an edge.

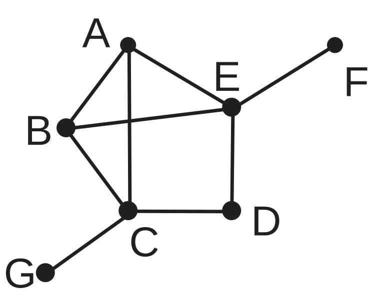
What are the pairs of adjacent vertices in the graph above?

A ***path*** is a route that passes from a vertex to an adjacent vertex with each vertex used being adjacent to the next vertex.

What are some paths that go from G to F?







An edge that is used more than once is ***deadheaded***.

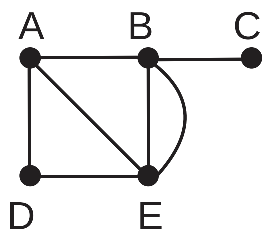
A path that uses every edge exactly once is an ***Euler path***.

A path that ends at the same vertex it started from is a ***circuit***.

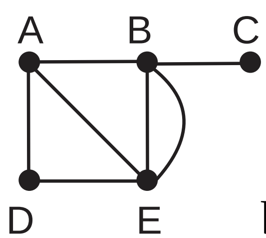
A circuit that uses every edge exactly once is an ***Euler circuit***.

<u>Example</u>

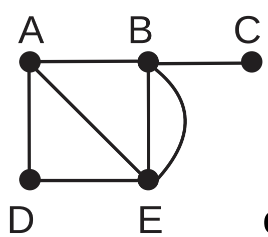
Classify the following paths for the graphs below:



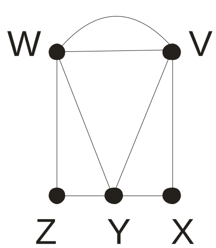
a) AEBC



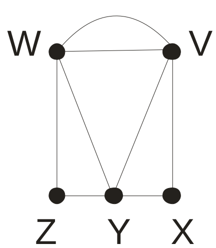
b) EDAEBE



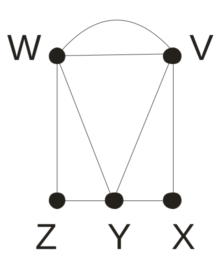
c) ADEABEBC



d) WVYZ



e) ZYWX

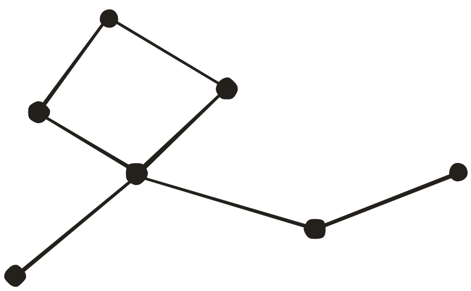


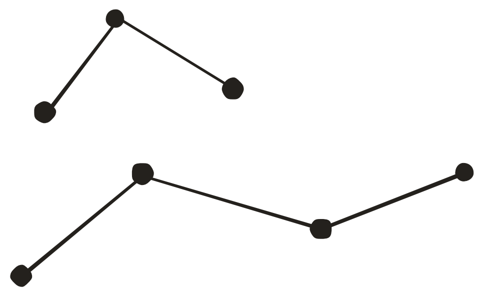
f) YWVXYZWVY

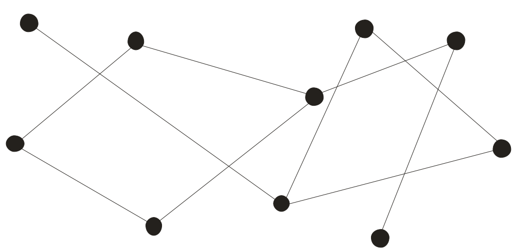
1.2 Finding Euler Circuits

The ***valence*** (or ***degree***) of a vertex is number of edges at that vertex.

A graph is ***connected*** if for every pair of vertices there is a path that connects them.





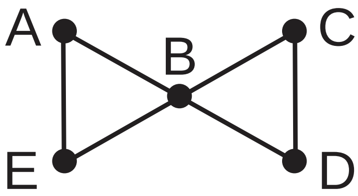


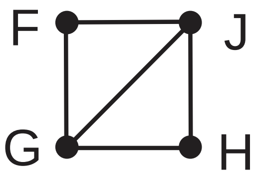
Euler's Theorem for a connected graph

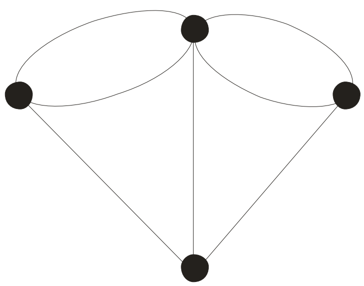
1. If the graph has no vertices of odd degree, then it has at least one Euler circuit and if a graph has an Euler circuit, then it has no vertices of odd degree.
2. If a graph has exactly 2 vertices of odd degree, then there is at least one Euler path, but no Euler circuit. Any Euler path must start at a vertex with an odd degree and end at the other vertex of odd degree.
3. If the graph has more than two vertices of odd degree, then it does not have an Euler path.

Example

Find an Euler circuit, if one exists



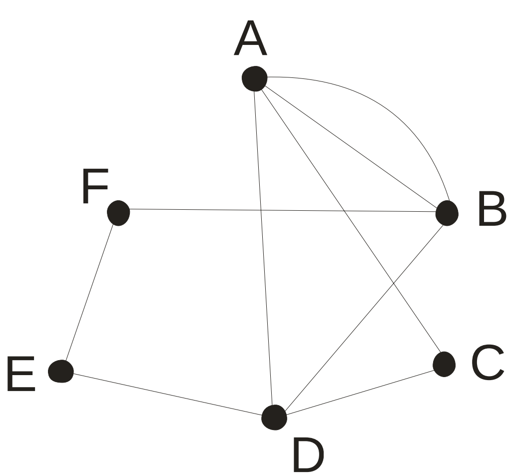




Some advice: Never use an edge that is the only link between two parts of a graph that still need to be covered.

Example

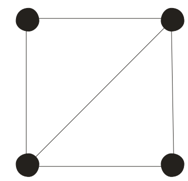
Find the Euler circuit for the graph below



1.3 Beyond Euler Circuits

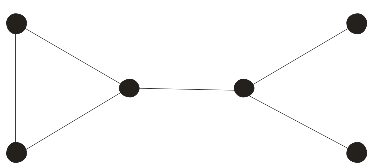
The ***Chinese Postman Problem***: How can we cover all edges with a minimum length path? If the graph has an Euler circuit, then that is the minimum length path.

If the graph has vertices with odd valence, repeat edges in such a way that there are no odd valence vertices. This ***Eulerizes*** the graph.



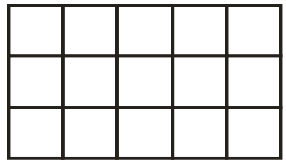
After finding the Euler circuit on the new graph you may ***squeeze*** the new graph onto the old graph by indicating where an edge is used more than once. Or you can simply add the extra edges to indicate where an edge is to be used more than once

Example
Eulerize the graph below.



A network is ***rectangular*** if the network consists of a series of rectangular blocks that form a larger rectangle. A rectangular network can be Eulerized by using an "***edge walker***" to walk around the outer boundary of the large rectangle and adds an edge to each odd valent vertex that connects to the next vertex.

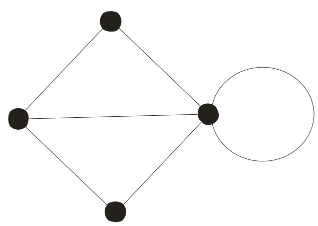
Example
Eulerize the following rectangular graphs using an edge walker.



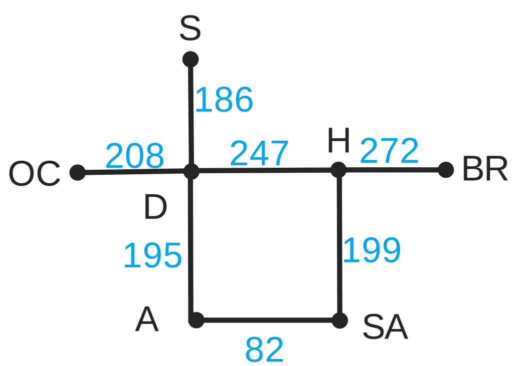
1.4 Urban Graph Traversal Problems

Mail delivery trucks need to go down both sides of a street (even if it is a one-way street), but a police patrol just needs to go down a street once.

Different edges can also have different "costs".



Example
How can you get from Baton Rouge to Austin?



Example
Eulerize the graph below at a minimum cost