Test 1—301 Answers

1. $y = e^{-\tan(x)} \displaystyle\int_{\pi/4}^{x} e^{\tan(u)} \cos(u) dx$

2. $y = \tan\left(\dfrac{1}{2} t e^{2t} - \dfrac{1}{4} e^{2t} + C \right)$

3. $x(0.1) \approx 0.1$, $x(0.2) \approx 0.16$.

4. (a) Linear, nonhomog. (b) Linear, nonhomog. (c) Nonlinear.

5. Let $L = D^2 - D - 6$, where $D = \frac{d}{dx}$.

   (a) $L[e^{rx}] = (r^2 - r + 6)e^{rx}$. $r_1 = -3$, $r_2 = 2$.
   (b) $W(e^{r_2 x}, e^{r_2 x}) = 0$. The set $\{e^{r_2 x}, e^{r_2 x}\}$ is linearly dependent.
   (c) $y = \dfrac{3}{5} e^{-3x} + \dfrac{2}{5} e^{2x}$

6. Concentration at time t is $c(t) = 0.2 - 0.19\,e^{-t/40}$.

7. The time at which the coffee is 120° is $t_0 = 10\dfrac{\ln(13/5)}{\ln(13/3)} \approx 6.5$ min.

8. $k = 2$, $\omega = 2\pi/\text{period} = 2\pi/24 = \pi/12$, $M(t) = 60 - 15\cos(\pi t/12)$.

$$T(t) = \frac{40 - \frac{5\pi^2}{12^2}}{4 + \frac{\pi^2}{12^2}} e^{-2t} - \frac{30}{4 + \frac{\pi^2}{12^2}} \left(2\cos\left(\frac{\pi t}{12}\right) + \frac{\pi}{12} \sin\left(\frac{\pi t}{12}\right) \right) + 60$$