Name_____ Seat # _____

Test I—Math 308-511

Part I: Short Answer

Instructions: *Place the answer in the space provided. Little or no partial credit will be given. You may use the back pages of your bluebook for scratch work; this work will NOT be examined. NO calculators that can graph, do algebra or do calculus are allowed.*

1. **(8 pts.)** What is the solution to $(x-1)y' - 3y = 1$, $y(2) = 1$? On what interval is it valid?

Answer: $y =$ Interval:_____

2. **(6 pts.)** What is the general solution to $xy' = (1-y^2)^{1/2}$?

Answer: $y =$

3. What is the general solution to each of these equations:

   (a) (6 pts.) $y'' - 2y' + 2y = 0$

Answer: $y =$

   (b) (6 pts.) $y'' + 9y = 0$

Answer: $y =$

   (c) (6 pts.) $y'' - 6y' + 9y = 0$

Answer: $y =$

4. **(8 pts.)** What is the form of the particular solution that you would get by using the method of undetermined coefficients to solve the inhomogeneous differential equation below? (The homogeneous solutions are the same as the ones for question 3(a).) **Do NOT find the coefficients.**

$$y'' - 2y' + 2y = xe^x \cos x + 3e^{4x}$$

Answer: $y_p =$

5. (8 pts.) A 16 lb weight stretches a spring $\frac{2}{5}$ ft. The weight is attached to a dashpot mechanism with a damping constant of 4 lb-sec/ft. What is the initial value problem for the system, if the weight is released from its equilibrium position with a velocity of $\frac{1}{4}$ ft/sec in the downward (positive) direction? Is the motion underdamped, critically damped, or overdamped? **DO NOT solve the initial value problem.**

Answer:	
Equation:	
$u(0) =$	
$u'(0) =$	
Damping:	

Part II: Essay Questions

Instructions: *Show all work in your bluebook, starting at the front of the bluebook.*

1. (15 pts.) A tank contains 100 gal of a solution made by dissolving 60 lb of salt in water. Salt water containing 1 lb of salt per gal runs in at the rate of 2 gal/min and the well-stirred solution runs out of the tank at the same rate. Solve the initial value problem involved to find the amount of salt in the tank at any time $t > 0$.

2. Consider the initial value problem,

$$x^2y'' + x(x-2)y' - (x-2)y = 0, \quad y(1) = 1,\ y'(1) = -2.$$

(a) (10 pts.) Given that $y_1 = x$ solves the homogeneous equation involved, use reduction of order to find a second solution.

(b) (5 pts.) Verify that the two solutions that you have found are linearly independent.

(c) (5 pts.) Solve the initial value problem.

3. (17 pts.) Use variation of parameters to find a particular solution to $y'' + 9y = 9\sec^2 3x$. (The homogeneous solutions are the same as the ones for question 3(b). To receive credit, you must derive the equations for u_1' and u_2' from first principles.)