A2. Table of Fourier Transforms

	$f(x) = \mathcal{F}^{-1}[F](x)$	$F(\omega) = \mathcal{F}[f](\omega) = \hat{f}(\omega)$
1	$f'(x)$	$-i\omega F(\omega)$
2	$f''(x)$	$-\omega^2 F(\omega)$
3	$f(ax+b) \quad (a>0)$	$\frac{1}{a} e^{-i(b/a)\omega} F(\omega/a)$
4	$(f*g)(x)$	$F(\omega)G(\omega)$
5	$\delta(x)$	$\frac{1}{\sqrt{2\pi}}$
6	$e^{iax} f(x)$	$F(\omega+a)$
7	$e^{-a^2x^2}$	$\frac{1}{\sqrt{2}a} e^{-\omega^2/(4a^2)}$
8	$xe^{-a^2x^2} \quad (a>0)$	$\frac{i}{2\sqrt{2}a^3} \omega e^{-\omega^2/(4a^2)}$
9	$x^2 e^{-a^2x^2} \quad (a>0)$	$\frac{1}{4\sqrt{2}a^5}(2a^2-\omega^2) e^{-\omega^2/(4a^2)}$
10	$\frac{1}{x^2+a^2} \quad (a>0)$	$\sqrt{\frac{\pi}{2}} \frac{1}{a} e^{-a\lvert\omega\rvert}$
11	$\frac{x}{x^2+a^2} \quad (a>0)$	$-i\sqrt{\frac{\pi}{2}} \frac{1}{2a} \omega e^{-a\lvert\omega\rvert}$
12	$H(a-\lvert x\rvert) = \begin{cases} 1, & \lvert x\rvert \le a \\ 0, & \lvert x\rvert > a \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin(a\omega)}{\omega}$
13	$xH(a-\lvert x\rvert) = \begin{cases} x, & \lvert x\rvert \le a \\ 0, & \lvert x\rvert > a \end{cases}$	$i\sqrt{\frac{2}{\pi}} \frac{1}{\omega^2} [\sin(a\omega) - a\omega\cos(a\omega)]$
14	$e^{-a\lvert x\rvert}$	$\sqrt{\frac{2}{\pi}} \frac{a}{a^2+\omega^2}$
15	$e^{-(x+b)^2/(4a)} + e^{-(x-b)^2/(4a)}$	$2\sqrt{2a} e^{-a\omega^2} \cos(b\omega)$
16	$\operatorname{erf}(ax)$	$i\sqrt{\frac{2}{\pi}} \frac{1}{\omega} e^{-\omega^2/(4a^2)}$
17	$x f(x)$	$-i \frac{d\hat{f}}{d\omega}$
18	$\hat{f}(x)$	$f(-\omega)$

Definitions

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{+i\omega x} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{-i\omega x} d\omega$$

Integrals

1. $\int e^{at} dt = \frac{1}{a} e^{at} + C$
2. $\int t^n e^{at} dt = \frac{1}{a} t^n e^{at} - \frac{n}{a} \int t^{n-1} e^{at} dt$
3. $\int t \sin(t) dt = \sin(t) - t\cos(t) + C$
4. $\int t \cos(t) dt = \cos(t) + t\sin(t) + C$
5. $\int e^{at} \cos(bt) dt = \frac{e^{at}}{a^2+b^2} (a\cos(bt) + b\sin(bt)) + C$
6. $\int e^{at} \sin(bt) dt = \frac{e^{at}}{a^2+b^2} (a\sin(bt) - b\cos(bt)) + C$
7. $\int \cos(at)\cos(bt) dt = \frac{\sin((a+b)t)}{2(a+b)} + \frac{\sin((a-b)t)}{2(a-b)} + C, \ a \neq b$
8. $\int \sin(at)\sin(bt) dt = \frac{\sin((a+b)t)}{2(a+b)} - \frac{\sin((a-b)t)}{2(a-b)} + C, \ a \neq b$
9. $\int \sin(at)\cos(bt) dt = -\frac{\cos((a+b)t)}{2(a+b)} - \frac{\cos((a-b)t)}{2(a-b)} + C, \ a \neq b$