Math 668 Lab

Multiresolution Analysis

Background. Here, we let $\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$. Recall that the Sampling Theorem states that if f is band-limited, with $\operatorname{supp}(\hat{f}) \subseteq [-\pi\nu, \pi\nu]$, then

$$f(t) = \sum_{n\in\mathbb{Z}} f\left(\frac{n}{\nu}\right) \operatorname{sinc}(\nu t - n)$$

The sampling spaces are taken to be

$$V_j = \{f \in L^2(\mathbb{R}) : \operatorname{supp}(\hat{f}) \subseteq [-2^{-j}\pi, 2^{-j}\pi]\},\ j \in \mathbb{Z},$$

and that they are nested, so $V_{j+1} \subset V_j$, $j \in \mathbb{Z}$.

1. Let $y = [a_1, a_2, a_3, \ldots, a_N]$ be a row vector. Write M-files that do the following:
   (a) Changes y to $Y_1 = [a_1, -a_2, a_3, \ldots, (-1)^N a_N]$, without multiplying with $(-1)^n$.
   (b) Changes y to $Y_2 = [a_N, -a_{N-1}, a_{N-2}, \ldots, (-1)^N a_1]$.
   (c) If N is even, use random numbers to verify that y and Y_2 are orthogonal, but that this is false when N is odd.

2. We want to use MATLAB to numerically verify that

$$\operatorname{sinc}\left(\frac{n}{2}\right) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \text{ is even and } n \neq 0 \\ \frac{(-1)^{\frac{n+1}{2}}}{\pi n} & \text{if } n \text{ is odd.} \end{cases}$$

   Write MATLAB code that just computes the sequence on the right for $n = -32$ to $n = 31$; plot it. Also plot the left side for the same vales of n.

3. Use the M-file that you made in exercise 1b along with the "Sampling Theorem" M-file that you made earlier to display the "sampling wavlet"

$$\psi(t) = \sum_{n\in\mathbb{Z}} (-1)^n \operatorname{sinc}\left(\frac{1-n}{2}\right) \operatorname{sinc}(2t - n)$$

4. Find the FFTs for both $\mathrm{sinc}(t)$ and $\psi(t)$. Use `subplot` to display the absolute values of the coefficients from each in a single figure. Give them titles, and label axes on both plots.

5. Use the FFT in MATLAB to verify that function $f(t) = \mathrm{sinc}^2(t)$ is in V_{-1}. The projection of f onto the space V_0 is

$$f_0(t) = \sum_{n \in \mathbb{Z}} (f * \mathrm{sinc})(n)\mathrm{sinc}(t - n).$$

   Find $(f*\mathrm{sinc})(n)$ for $n = -64$ to $n = 63$. Plot f_0 using these coefficients.

6. The space wavelet space at level 0 is

$$W_0 = \{g \in V_{-1} : \int_{\mathbb{R}} g(t)\overline{f(t)}dt = 0 \ \forall f \in V_0\}$$

   Using exercise 5, verify that $g_0 = f - f_0$ is the "wavelet part" of f, and plot g_0.

Due 12 November 1998