Test II

Instructions: *Show all work in your bluebook. Calculators that do linear algebra or calculus are* not *allowed.*

1. Define each space listed and describe the operations of vector addition (+) and scalar multiplication (·) corresponding to it.

   (a) **(5 pts.)** $\mathcal{P}_n$

   (b) **(5 pts.)** $C^{(1)}[0,1]$

2. **(15 pts.)** Determine whether or not the set S of 2×2 matrices $M = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$ such that $x + w = 0$ is a subspace of $\mathcal{M}_{2,2}$.

3. **(15 pts.)** Determine whether or not the set $\{1, e^x, e^{2x}\}$ is linearly independent in $C(-\infty, \infty)$.

4. **(10 pts.)** Consider $G : C(-\infty,\infty) \to C(-\infty,\infty)$ given by $Gu(x) = \int_0^x e^t u(t)dt$. Show that G is linear and that it is one-to-one.

5. **(20 pts.)** Find bases for the column space, null space, and row space of C, and state the rank and nullity of C. What should these sum to? Do they?

$$C = \begin{pmatrix} 1 & -3 & -1 & -3 \\ -1 & 3 & 2 & 4 \\ 2 & -6 & 4 & 0 \end{pmatrix}$$

6. Given that $L : \mathcal{P}_2 \to \mathcal{P}_2$ be defined by $L(p) = x^2 p'' - 2(x-1)p' + 3p$ is a linear transformation, do the following:

   (a) **(10 pts.)** Find the matrix of L relative to the basis $B = \{1, x, x^2\}$.

   (b) **(5 pts.)** Find $[2 - x + x^2]_B$ and use the matrix from part 6a to solve $L(p) = 2 - x + x^2$ for p.

7. Let $A = \begin{pmatrix} 2 & -1 \\ 2 & 5 \end{pmatrix}$.

   (a) **(10 pts.)** Find the eigenvalues and eigenvectors of A.

   (b) **(5 pts.)** Use the answer to part 7a to solve $\dfrac{d\mathbf{x}}{dt} = A\mathbf{x}$.