Quiz - Assignment 5

Instructions: In questions 1 to 3, state what each space is and describe the operations of vector addition (+) and scalar multiplication ($\cdot$) corresponding to it.

1. **(10 pts.)** $\mathcal{P}_n$ is the set of all polynomials of degree n or less; that is, $\mathcal{P}_n = \{a_0 + a_1x + \cdots + a_nx^n\}$. Here are the operations. If $p, q \in \mathcal{P}$, $p(x) = a_0 + a_1x + \cdots + a_nx^n$ $q(x) = b_0 + a_1x + \cdots + b_nx^n$, then $(p+q)(x) = (a_0 + b_0) + (a_1 + b_1)x + \cdots + (a_n + b_n)x^n$.
   If c is a scalar, then $c \cdot p$ is the polynomial
   $(c \cdot p)(x) = ca_0 + ca_1x + \cdots + ca_nx^n$.

2. **(10 pts.)** $C[a, b]$ is the set of all functions f defined and continuous on the interval $[a, b]$. If $f, g \in C[a, b]$, then $f + g$ is defined by
   $(f + g)(x) = f(x) + g(x)$
   and $c \cdot f$ is defined by
   $(c \cdot f)(x) = cf(x)$.

3. **(10 pts.)** $\mathcal{M}_{m,n}$ is the set of all $m \times n$ matrices. If $A, B \in \mathcal{M}_{m,n}$, then $A + B$ is ordinary matrix addition, and if c is a scalar and $A \in \mathcal{M}_{m,n}$, then $c \cdot A$ is ordinary multiplication of a matrix by a scalar.

4. **(10 pts.)** *Define the term* subspace. A nonempty subset $\mathcal{V}$ of a vector space $\mathcal{W}$ is a subspace of $\mathcal{W}$ is $\mathcal{V}$ is closed under the operations of + and $\cdot$ from $\mathcal{W}$.

5. **(10 pts.)** *Let* $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$ *be a set of vectors in a vector space* $\mathcal{V}$. *Define* span(S). The span(S) is the set of all linear combinations of the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$. Equivalently,

$$\mathrm{span}(S) = \{c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_m\mathbf{v}_m \,|\, c_1, \cdots, c_m \text{ are scalars}\}.$$