Assignment 2—Math 668

1. Let h be a continuous, L^1 impulse response function. Show that h is causal if and only if $h(t) \equiv 0$ for all $t < 0$.

2. The *support* of a complex-valued function f is the closure of the set of all points for which $f \neq 0$. We denote this by $\mathrm{supp}(f)$. We say that f is *compactly supported* or has *compact support* if $\mathrm{supp}(f)$ is a compact set. (In $\mathbb{R}^n$, this means closed and bounded.) Find the support of the following functions. Which are compactly supported?

$$f(x) := \begin{cases} 1 & \text{if } k < x < k+1 \text{ for } k \text{ odd} \\ 0 & \text{if } k \leq x \leq k+1 \text{ for } k \text{ even} \end{cases} \tag{1}$$

$$g(x) := \begin{cases} x(1-x) & \text{if } 0 \leq x \leq 3 \\ 1 & \text{if } x > 3 \\ -1 & \text{if } -5 < x < 0 \\ 0 & \text{if } x \leq -5 \end{cases} \tag{2}$$

$$h(r,\theta) := \begin{cases} r^2(1-r^2) & \text{if } 0 \leq r < 1 \text{ and } 0 \leq \theta < 2\pi \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

3. Suppose that we are given a family of funtions $f_\delta : \mathbb{R} \to \mathbb{R}$ that depend on a parameter $\delta > 0$ and satisfy these properties: $f_\delta \in L^1(\mathbb{R})$, $\int_{\mathbb{R}} f_\delta(x)dx = 1$, $f_\delta \geq 0$, $\mathrm{supp}(f) \subset [-\delta, \delta]$. Show that if h is a continuous, L^1 response funtion, then

$$\lim_{\delta \downarrow 0} f_\delta * h(t) = h(t).$$

4. Give several examples of f_δ-families. For a specific "impulse family", do plots illustrating that $f_\delta * h(t) \to h(t)$ as $\delta \downarrow 0$. (Pick a simple family!)

5. Plot the time-frequency rectangle (cf. Mallat, pgs. 3-6) for the function

$$f(t) := \begin{cases} t & \text{if } 0 \leq t \leq 1 \\ 2-t & \text{if } 1 < t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Due Thursday, October 1.