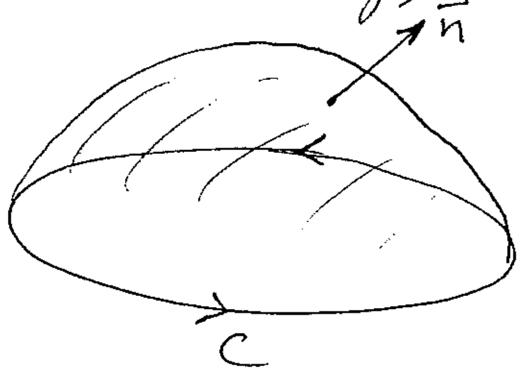
<u>Stokes's Theorem</u>: Let S be a piecewise smooth oriented surface whose boundary C is a piecewise smooth simple closed curve, which is directed in accordance with the orientation of S. If $\vec{F}(x,y,z)$ is a vector field whose components are continuous and have continuous first partials, then

$$\oint_C \vec{F}\cdot d\vec{x} = \iint_S (\nabla\times\vec{F})\cdot\vec{n}\,dS$$

<u>Examples</u>: ① Verify Stokes's theorem for $\vec{F} = 2y\vec{i} + 3x\vec{j} - z^3\vec{k}$, where S is the upper half of $x^2+y^2+z^2=9$ and C is its boundary; the orientations are shown.



$[0 \le \varphi \le \pi/2,\ 0 \le \theta \le 2\pi]$

Line integral: $$\oint_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} 2\cdot 3^2(\sin(t))(-\sin(t)) + 3^3\cos^2(t)\,dt$$

$$= 9\int_0^{2\pi}\left[3\cos^2(t) - 2\sin^2(t)\right]dt$$

$$= 9\left[3\pi - 2\pi\right] = 9\pi$$

Surface Integral

$$\int_S \underbrace{\nabla\times\vec{F}}_{\vec{k}}\cdot d\vec{S} = \iint_S \vec{k}\cdot\vec{n}\,dS = \int_0^{\frac{\pi}{2}}\int_0^{2\pi}\cos\varphi\,\underbrace{(3)^2\sin\varphi\,d\theta\,d\varphi}_{\text{Hemisphere}}$$

$$= 9\cdot 2\pi\int_0^{\frac{\pi}{2}}\frac{d}{d\varphi}\left[\tfrac{1}{2}\sin^2\varphi\right] = 9\pi.$$