# HWK 10.1

Thursday, April 7, 2022 9:09 PM

#31. Compute $\int_X yz\,dx - xz\,dy + xy\,dz$

$X$: connecting $(1,1,2) \to (5,3,1)$.

$X(t) = (1,1,2) + t\big((5,3,1) - (1,1,2)\big); \quad 0 \le t \le 1$

$= (1,1,2) + t(4,2,-1) = (1+4t, 1+2t, 2-t)$

or $x(t) = 1+4t$

$y(t) = 1+2t$

$z(t) = 2-t$.

then

$$\int_X yz\,dx - xz\,dy + xy\,dz$$

$$= \int_0^1 \left[(1+2t)(2-t)4 - (1+4t)(2-t)2 + (1+4t)(1+2t)\right] dt$$

$$= \int_0^1 (-8t^2 - 8t + 5)\,dt = \frac{-8}{3} - \frac{8}{2} + 5$$

#27 $y = x^2$ $\quad X: (3,9) \to (0,0)$

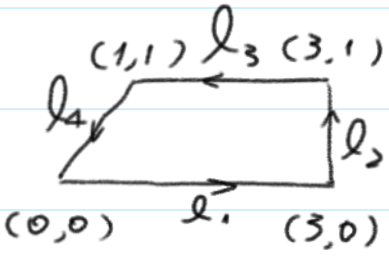
$$\int_X y\,dx - x\,dy$$

set $x = t$, $y = t^2$, $\quad t\Big|_3^0$

$$\int_X y\,dx - x\,dy = \int_3^0 (t^2 - t \cdot 2t)\,dt$$

$$= \int_3^0 -t^2\,dt = -\frac{t^3}{3}\Big|_3^0 = 9.$$

#25 Compute $\int_X x^2y\,dx - (x+y)\,dy$




$$= \int_{l_1} + \int_{l_2} + \int_{l_3} + \int_{l_4} (x^2y\,dx - (x+y)\,dy$$

$$= 0 + \int_0^1 (3+y)\,dy + \int_3^1 x^2\,dx + \int_1^0 (x^3 - 2x)\,dx$$

$$= 3(3-1) + \frac{y^2}{2}\Big|_0^1 + \frac{x^3}{3}\Big|_3^1 + \frac{x^4}{4}\Big|_1^0 - x^2\Big|_1^0$$

$l_1$: $y = 0$, $dy = 0$, $x\Big|_0^3$

$l_2$: $x = 3$, $dx = 0$, $y\Big|_0^1$

$l_3$: $y = 1$, $dy = 0$, $x\Big|_3^1$

$l_4$: $x = y\Big|_1^0$

#23. $F = (2z^5 - 3xy)\hat{i} - x^2\hat{j} + x^2z\hat{k}$

Compute $\int_X F(x(t)) \cdot x'(t)\,dt$. Use

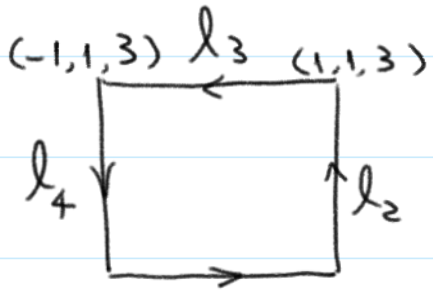
$$\int_X M\,dx + N\,dy + P\,dz, \quad z = 3,\ dz = 0$$

$$= \int_{l_1} M\,dx + \int_{l_2} N\,dy + \int_{l_3} M\,dx + \int_{l_4} N\,dy$$

$$= \int_{-1}^{1} (2\cdot 3^5 - 3x(-1))\,dx + \int_{-1}^{1} -1\,dy$$

$$+ \int_{1}^{-1} (2\cdot 3^5 - 3x\cdot 1)\,dx + \int_{1}^{-1} -1\,dy$$

$= 0$




$l_1$: $x|_{-1}^{1}$, $y = -1$, $dy = 0$

$l_2$: $x = 1$, $dx = 0$, $y|_{-1}^{1}$

$l_3$: $x|_{1}^{-1}$, $y = 1$, $dy = 0$.

$l_4$: $x = -1$, $dx = 0$, $y|_{1}^{-1}$

$x$ (odd)