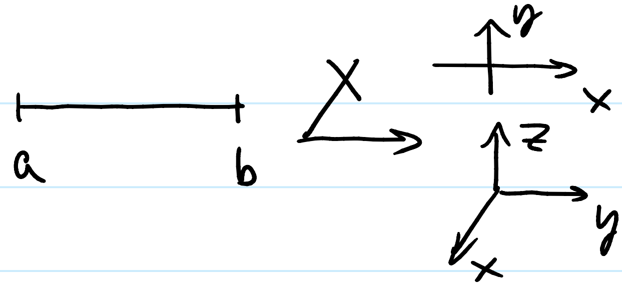
Add to § 7-8

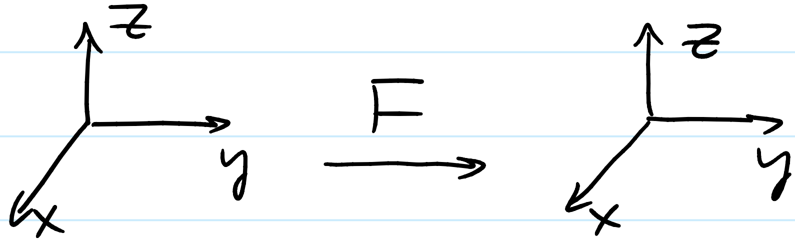
Friday, June 22, 2018 2:27 AM

Path: $X(t) = \begin{matrix} (x(t), y(t)) \\ (x(t), y(t), z(t)) \end{matrix}$.

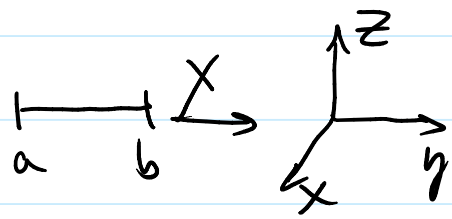


Vector field: $F(x,y,z) = (F_1(x,y,z), F_2(x,y,z), F_3(x,y,z))$

$= (M(x,y,z), N(x,y,z), P(x,y,z))$



$X(t) = (x(t), y(t), z(t)) = \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$,



Vector field.

$F = y\,i - x\,j$

$F = (x \sin y, y \cos x)$, $-2\pi \le x, y \le 2\pi$.

$F = (y^3 x, x^2 y)$, $-1 \le x \le 1$, $-2 \le y \le 2$.

$G = 2x\,i + 2y\,j - 3k$

$G = 2x\,i + 2y\,j - 3k$

is a gradient field, since for

$F = x^2 + y^2 - 3z$

we have

$\nabla F = (F_x, F_y, F_z) = (2x, 2y, -3) = G.$

For a vector field F

$\nabla \cdot F = 0$ incompressive

$\nabla \times F = 0$ irrotational

Ex. $F = x^2y\,i + xz\,j + xyz\,k = (M, N, P)$

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

$$= 2xy + 0 + xy = 3xy$$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2y & xz & xyz & x^2y & xz \end{array}\right|$$

$$= xz\,i + 0\,j + z\,k - x^2\,k - x\,i - yz\,j$$

$$= (xz - x)\,i - yz\,j + (z - x^2)\,k$$

Ex $F = (3x^2z + y^2)\hat{i} + 2xy\,\hat{j} + (x^3 - 2z)\hat{k}$

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial}{\partial x}(3x^2z + y^2) + \frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(x^3 - 2z)$$

$$= 6xz + 2x + (-2)$$

$$\operatorname{Curl} F = \nabla \times F = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 3x^2z + y^2 & 2xy & x^3 - 2z & 3x^2z + y^2 & 2xy \end{array} \right|$$

$$= 0\hat{i} + 3x^2\hat{j} + 2y\hat{k} - 2y\hat{k} - 0\hat{i} - 3x^2\hat{j} = 0$$

irrotational!