(f) $\left[\begin{array}{rrr|r} 1 & -1 & 3 & 8 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

3. The augmented matrices that follow are in reduced row echelon form. In each case, find the solution set of the corresponding linear system.

(a) $\left[\begin{array}{rrr|r} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$ (b) $\left[\begin{array}{rrr|r} 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

(c) $\left[\begin{array}{rrr|r} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{rrrr|r} 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 & 4 \end{array}\right]$

(e) $\left[\begin{array}{rrrr|r} 1 & 5 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(f) $\left[\begin{array}{rrr|r} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

4. For each of the systems in Exercise 3, make a list of the lead variables and a second list of the free variables.

5. For each of the systems of equations that follow, use Gaussian elimination to obtain an equivalent system whose coefficient matrix is in row echelon form. Indicate whether the system is consistent. If the system is consistent and involves no free variables, use back substitution to find the unique solution. If the system is consistent and there are free variables, transform it to reduced row echelon form and find all solutions.

(a) $\begin{aligned} x_1 - 2x_2 &= 3 \\ 2x_1 - x_2 &= 9 \end{aligned}$ (b) $\begin{aligned} 2x_1 - 3x_2 &= 5 \\ -4x_1 + 6x_2 &= 8 \end{aligned}$

(c) $\begin{aligned} x_1 + x_2 &= 0 \\ 2x_1 + 3x_2 &= 0 \\ 3x_1 - 2x_2 &= 0 \end{aligned}$

(d) $\begin{aligned} 3x_1 + 2x_2 - x_3 &= 4 \\ x_1 - 2x_2 + 2x_3 &= 1 \\ 11x_1 + 2x_2 + x_3 &= 14 \end{aligned}$

(e) $\begin{aligned} 2x_1 + 3x_2 + x_3 &= 1 \\ x_1 + x_2 + x_3 &= 3 \\ 3x_1 + 4x_2 + 2x_3 &= 4 \end{aligned}$

(f) $\begin{aligned} x_1 - x_2 + 2x_3 &= 4 \\ 2x_1 + 3x_2 - x_3 &= 1 \\ 7x_1 + 3x_2 + 4x_3 &= 7 \end{aligned}$

(g) $\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_1 + 3x_2 - x_3 - x_4 &= 2 \\ 3x_1 + 2x_2 + x_3 + x_4 &= 5 \\ 3x_1 + 6x_2 - x_3 - x_4 &= 4 \end{aligned}$

(h) $\begin{aligned} x_1 - 2x_2 &= 3 \\ 2x_1 + x_2 &= 1 \\ -5x_1 + 8x_2 &= 4 \end{aligned}$

(i) $\begin{aligned} -x_1 + 2x_2 - x_3 &= 2 \\ -2x_1 + 2x_2 + x_3 &= 4 \\ 3x_1 + 2x_2 + 2x_3 &= 5 \\ -3x_1 + 8x_2 + 5x_3 &= 17 \end{aligned}$

(j) $\begin{aligned} x_1 + 2x_2 - 3x_3 + x_4 &= 1 \\ -x_1 - x_2 + 4x_3 - x_4 &= 6 \\ -2x_1 - 4x_2 + 7x_3 - x_4 &= 1 \end{aligned}$

(k) $\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\ x_1 - 5x_2 + x_4 &= 5 \end{aligned}$

(l) $\begin{aligned} x_1 - 3x_2 + x_3 &= 1 \\ 2x_1 + x_2 - x_3 &= 2 \\ x_1 + 4x_2 - 2x_3 &= 1 \\ 5x_1 - 8x_2 + 2x_3 &= 5 \end{aligned}$

6. Use Gauss–Jordan reduction to solve each of the following systems:

(a) $\begin{aligned} x_1 + x_2 &= -1 \\ 4x_1 - 3x_2 &= 3 \end{aligned}$

(b) $\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\ 3x_1 + x_2 + 2x_3 - x_4 &= -1 \end{aligned}$

(c) $\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1 - x_2 - x_3 &= 0 \end{aligned}$

(d) $\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_1 + x_2 - x_3 + 3x_4 &= 0 \\ x_1 - 2x_2 + x_3 + x_4 &= 0 \end{aligned}$

7. Give a geometric explanation of why a homogeneous linear system consisting of two equations in three unknowns must have infinitely many solutions. What are the possible numbers of solutions of a nonhomogeneous 2×3 linear system? Give a geometric explanation of your answer.