SECTION 2 EXERCISES

1. Evaluate each of the following determinants by inspection:

(a) $\begin{vmatrix} 0 & 0 & 3 \\ 0 & 4 & 1 \\ 2 & 3 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ -1 & -1 & -1 & 2 \end{vmatrix}$

(c) $\begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$

2. Let

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ -2 & -2 & 3 & 3 \\ 1 & 2 & -2 & -3 \end{bmatrix}.$$

(a) Use the elimination method to evaluate $\det(A)$.

(b) Use the value of $\det(A)$ to evaluate

$$\begin{vmatrix} 0 & 1 & 2 & 3 \\ -2 & -2 & 3 & 3 \\ 1 & 2 & -2 & -3 \\ 1 & 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & 4 & 4 \\ 2 & 3 & -1 & -2 \end{vmatrix}$$

3. For each of the following, compute the determinant and state whether the matrix is singular or nonsingular:

(a) $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 5 \\ 2 & 1 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & -2 \\ 1 & 4 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 7 & 3 \end{bmatrix}$

4. Find all possible choices of c that would make the following matrix singular:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{bmatrix}$$

5. Let A be an $n \times n$ matrix and α a scalar. Show that

$$\det(\alpha A) = \alpha^n \det(A)$$

6. Let A be a nonsingular matrix. Show that

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

7. Let A and B be 3×3 matrices with $\det(A) = 4$ and $\det(B) = 5$. Find the value of

(a) $\det(AB)$ (b) $\det(3A)$

(c) $\det(2AB)$ (d) $\det(A^{-1}B)$

8. Show that if E is an elementary matrix, then E^T is an elementary matrix of the same type as E.

9. Let E_1, E_2, and E_3 be 3×3 elementary matrices of types I, II, and III, respectively, and let A be a 3×3 matrix with $\det(A) = 6$. Assume, additionally, that E_2 was formed from I by multiplying its second row by 3. Find the values of each of the following:

(a) $\det(E_1 A)$ (b) $\det(E_2 A)$

(c) $\det(E_3 A)$ (d) $\det(AE_1)$

(e) $\det(E_1^2)$ (f) $\det(E_1 E_2 E_3)$

10. Let A and B be row equivalent matrices, and suppose that B can be obtained from A by using only row operations I and III. How do the values of $\det(A)$ and $\det(B)$ compare? How will the values compare if B can be obtained from A by using only row operation III? Explain your answers.

11. Let A be an $n \times n$ matrix. Is it possible for $A^2 + I = O$ in the case where n is odd? Answer the same question in the case where n is even.

12. Consider the 3×3 Vandermonde matrix

$$V = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$$

(a) Show that $\det(V) = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)$. [*Hint*: Make use of row operation III.]

(b) What conditions must the scalars x_1, x_2, and x_3 satisfy in order for V to be nonsingular?