SECTION 2 EXERCISES

1. Determine whether the following sets form subspaces of $\mathbb{R}^2$:
   (a) $\{(x_1, x_2)^T \mid x_1 + x_2 = 0\}$
   (b) $\{(x_1, x_2)^T \mid x_1 x_2 = 0\}$
   (c) $\{(x_1, x_2)^T \mid x_1 = 3x_2\}$
   (d) $\{(x_1, x_2)^T \mid |x_1| = |x_2|\}$
   (e) $\{(x_1, x_2)^T \mid x_1^2 = x_2^2\}$
2. Determine whether the following sets form subspaces of $\mathbb{R}^3$:
   (a) $\{(x_1, x_2, x_3)^T \mid x_1 + x_3 = 1\}$
   (b) $\{(x_1, x_2, x_3)^T \mid x_1 = x_2 = x_3\}$
   (c) $\{(x_1, x_2, x_3)^T \mid x_3 = x_1 + x_2\}$
   (d) $\{(x_1, x_2, x_3)^T \mid x_3 = x_1 \text{ or } x_3 = x_2\}$
3. Determine whether the following are subspaces of $\mathbb{R}^{2\times 2}$:
   (a) The set of all 2×2 diagonal matrices
   (b) The set of all 2×2 triangular matrices
   (c) The set of all 2×2 lower triangular matrices
   (d) The set of all 2×2 matrices A such that $a_{12} = 1$
   (e) The set of all 2×2 matrices B such that $b_{11} = 0$
   (f) The set of all symmetric 2×2 matrices
   (g) The set of all singular 2×2 matrices
4. Determine the null space of each of the following matrices:
   (a) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & 2 & -3 & -1 \\ -2 & -4 & 6 & 3 \end{pmatrix}$
   (c) $\begin{pmatrix} 1 & 3 & -4 \\ 2 & -1 & -1 \\ -1 & -3 & 4 \end{pmatrix}$
   (d) $\begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -1 & 0 & -5 \end{pmatrix}$
5. Determine whether the following are subspaces of P_4 (be careful!):
   (a) The set of polynomials in P_4 of even degree
   (b) The set of all polynomials of degree 3
   (c) The set of all polynomials $p(x)$ in P_4 such that $p(0) = 0$
   (d) The set of all polynomials in P_4 having at least one real root
6. Determine whether the following are subspaces of $C[-1, 1]$:
   (a) The set of functions f in $C[-1, 1]$ such that $f(-1) = f(1)$
   (b) The set of odd functions in $C[-1, 1]$
   (c) The set of continuous nondecreasing functions on $[-1, 1]$
   (d) The set of functions f in $C[-1, 1]$ such that $f(-1) = 0$ and $f(1) = 0$
   (e) The set of functions f in $C[-1, 1]$ such that $f(-1) = 0$ or $f(1) = 0$
7. Show that $C^n[a, b]$ is a subspace of $C[a, b]$.
8. Let A be a fixed vector in $\mathbb{R}^{n\times n}$ and let S be the set of all matrices that commute with A; that is,

$$S = \{B \mid AB = BA\}$$

Show that S is a subspace of $\mathbb{R}^{n\times n}$.